Honors Precalculus - Unit 3

Name KEY

Unit 3 Review

Date________ Period____

Use identities to find the value of each expression.

1) Find $\cos\theta$ and $\cot\theta$ if $\sec\theta = -\frac{7}{4}$ and $\sin\theta > 0$.

$\boxed{\cos = -\frac{4}{7}}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \tan^2\theta = \left(-\frac{7}{4}\right)^2$

$1 + \tan^2\theta = \frac{49}{16}$

$\tan^2\theta = \frac{33}{16}$

$\tan\theta = -\frac{\sqrt{33}}{4} \rightarrow$

$\cot\theta = -\frac{4}{\sqrt{33}} = \boxed{\frac{-4\sqrt{33}}{33}}$

2) Find $\tan\theta$ and $\sec\theta$ if $\cot\theta = -\frac{4}{5}$ and $\sec\theta < 0$.

$\boxed{\tan\theta = -\frac{5}{4}}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \left(-\frac{5}{4}\right)^2 = \sec^2\theta$

$1 + \frac{25}{16} = \sec^2\theta$

$\frac{41}{16} = \sec^2\theta$

$\boxed{-\frac{\sqrt{41}}{4} = \sec\theta}$

Verify each identity.

3) $\frac{1 - \csc x}{\csc x} = \sin x - 1$ GIVEN

$\frac{1}{\csc x} - \frac{\csc x}{\csc x} = \sin x - 1$ SUBTRACTION

$\sin x - 1 = \sin x - 1$ ■ QUOTIENT ID.

4) $\tan^2 x - \cot^2 x = \sec^2 x - \csc^2 x$ GIVEN

$\sec^2 x - 1 - \cot^2 x = \sec^2 x - \csc^2 x$ PYTHAG ID

$\sec^2 x - 1 - (\csc^2 x - 1) = \sec^2 x - \csc^2 x$ PYTHAG ID

$\sec^2 x - 1 - \csc^2 x + 1 = \sec^2 x - \csc^2 x$ DISTRIBUTE

$\sec^2 x - \csc^2 x = \sec^2 x - \csc^2 x$ ■ ADD/SUBT.

5) $\frac{\csc^2 x}{\tan^2 x} = \frac{\cot^2 x}{\sin^2 x}$ GIVEN

$\frac{1}{\tan^2 x} \cdot \csc^2 x = \frac{\cot^2 x}{\sin^2 x}$ DIVIDE

$\cot^2 x \cdot \csc^2 x = \frac{\cot^2 x}{\sin^2 x}$ RECIPROCAL ID

$\cot^2 x \cdot \frac{1}{\sin^2 x} = \frac{\cot^2 x}{\sin^2 x}$ RECIPROCAL ID

$\frac{\cot^2 x}{\sin^2 x} = \frac{\cot^2 x}{\sin^2 x}$ ■ MULTIPLICATION

6) $\csc^2 x + \sec^2 x = \dfrac{\csc^2 x}{\cos^2 x}$ GIVEN

$\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x} = \dfrac{\csc^2 x}{\cos^2 x}$ RECIPROCAL ID.

$\dfrac{\cos^2 x}{\sin^2 x\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x\sin^2 x} = \dfrac{\csc^2 x}{\cos^2 x}$ MULTIPLICATION

$\dfrac{\cos^2 x+\sin^2 x}{\sin^2 x\cos^2 x} = \dfrac{\csc^2 x}{\cos^2 x}$ ADDITION

$\dfrac{1}{\sin^2 x\cos^2 x} = \dfrac{\csc^2 x}{\cos^2 x}$ PYTHAG. ID

$\dfrac{\csc^2 x}{\cos^2 x} = \dfrac{\csc^2 x}{\cos^2 x}$ ■ RECIPROCAL ID.

7) $\cot x\sec^2 x\tan x = \tan^2 x + 1$ GIVEN

$\dfrac{1}{\tan x} \cdot \sec^2 x \cdot \dfrac{\tan x}{1} = \tan^2 x + 1$ RECIPROCAL ID

$\sec^2 x = \tan^2 x + 1$ MULTIPLICATION/ DIVISION

$\tan^2 x + 1 = \tan^2 x + 1$ ■ PYTHAGOREAN ID.

8) $\cot x\tan x - \sec^2 x = -\tan^2 x$ GIVEN

$\dfrac{1}{\tan x} \cdot \dfrac{\tan x}{1} - \sec^2 x = -\tan^2 x$ RECIPROCAL ID

$1 - \sec^2 x = -\tan^2 x$ MULTIPLICATION/ DIVISION

$-\tan^2 x = -\tan^2 x$ ■ PYTHAGOREAN ID.

9) $\dfrac{\tan x + \cot x}{\sec x} = \csc x$ GIVEN

$\dfrac{\tan x}{\sec x} + \dfrac{\cot x}{\sec x} = \csc x$ ADDITION

$\dfrac{\frac{\sin x}{\cos x}}{\sec x} + \dfrac{\frac{\cos x}{\sin x}}{\sec x} = \csc x$ QUOTIENT ID.

$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} + \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\cos x}} = \csc x$ RECIPROCAL ID.

$\dfrac{\sin x}{\cos x} \cdot \dfrac{\cos x}{1} + \dfrac{\cos x}{\sin x} \cdot \dfrac{\cos x}{1} = \csc x$ MULTIPL.

$\dfrac{\sin x}{1} + \dfrac{\cos^2 x}{\sin x} = \csc x$ DIVISION

$\dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x} = \csc x$ MULTIPL.

$\dfrac{1}{\sin x} = \csc x$ PYTHAGOREAN ID.

$\csc x = \csc x$ ■ RECIPROCAL ID.

PRODUCT TO SUM

Write each product as a sum or difference.

10) $2\cos 65°\cos 30°$

$= 2\left(\frac{1}{2}[\cos(65-30) + \cos(65+30)]\right)$

$= \boxed{\cos 35 + \cos 95}$

11) $\cos 30°\cos 65°$

$= \frac{1}{2}[\cos(30-65) + \cos(30+65)]$

$= \boxed{\frac{1}{2}(\cos -35 + \cos 95)}$

12) $-4\sin\theta\cos 7\theta$

$= -4\left(\frac{1}{2}[\sin(\theta+7\theta) + \sin(\theta-7\theta)]\right)$

$= \boxed{-2(\sin 8\theta + \sin -6\theta)}$

SUM TO PRODUCT

Write each sum or difference as a product.

13) $\cos 32° + \cos 112°$

$= 2\cos\left(\frac{32+112}{2}\right)\cos\left(\frac{32-112}{2}\right)$

$= 2\cos\left(\frac{144}{2}\right)\cos\left(\frac{-80}{2}\right)$

$= \boxed{2\cos 72 \cos -40}$

14) $\sin 13° - \sin 231°$

$= 2\cos\left(\frac{13+231}{2}\right)\sin\left(\frac{13-231}{2}\right)$

$= 2\cos\left(\frac{244}{2}\right)\sin\left(\frac{-218}{2}\right)$

$= \boxed{2\cos 122 \sin -109}$

15) $-4(\cos 15B - \cos 3B)$

$= -4\left[-2\sin\left(\frac{15B+3B}{2}\right)\sin\left(\frac{15B-3B}{2}\right)\right]$

$= +8\sin\left(\frac{18B}{2}\right)\sin\left(\frac{12B}{2}\right)$

$= \boxed{8\sin 9B \sin 6B}$

Use the sum and difference formulas to find the exact value of each.

16) $\sin 105°$ $\quad 60° + 45° = 105°$

$= \sin(60° + 45°)$

$= \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

17) $\sin \frac{11\pi}{12}$ $\quad \frac{3\pi}{4} + \frac{\pi}{6} \rightarrow \frac{9\pi}{12} + \frac{2\pi}{12} = \frac{11\pi}{12}$

$= \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \cos \frac{3\pi}{4} \sin \frac{\pi}{6}$

$= \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) + \left(\frac{-\sqrt{2}}{2} \cdot \frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{-\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

18) $\cos 15°$ $\quad 45° - 30° = 15°$

$= \cos(45° - 30°)$

$= \cos 45° \cos 30° + \sin 45° \sin 30°$

$= \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

19) $\cos \frac{\pi}{12}$ $\quad \frac{\pi}{3} - \frac{\pi}{4} \rightarrow \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$

$= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4}$

$= \left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$

20) $\tan 165°$ $\quad 120° + 45° = 165°$

$= \tan(120° + 45°)$

$= \frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°} = \frac{-\sqrt{3} + 1}{1 - (-\sqrt{3} \cdot 1)}$

$= \frac{-\sqrt{3} + 1}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{-\sqrt{3} + 3 + 1 - \sqrt{3}}{1 - \sqrt{3} + \sqrt{3} - 3}$

$= \frac{4 - 2\sqrt{3}}{-2} = \boxed{-2 + \sqrt{3}}$

21) $\tan \frac{\pi}{12}$ $\quad \frac{\pi}{3} - \frac{\pi}{4} \rightarrow \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$

$= \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{4}} = \frac{\sqrt{3} - 1}{1 + (\sqrt{3} \cdot 1)}$

$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - \sqrt{3} + \sqrt{3} - 3}$

$= \frac{-4 + 2\sqrt{3}}{-2} = \boxed{2 - \sqrt{3}}$