Honors Precalculus - Unit 3

Name KEY

Double Angles - Solving & Proving

Date________________ Period____

Solve each equation for $0 \le \theta < 2\pi$.

1) $3\sin 2\theta = 4\sin 2\theta + \sin\theta$

$0 = \sin 2\theta + \sin\theta$

$0 = 2\sin\theta\cos\theta + \sin\theta$

$0 = \sin\theta(2\cos\theta + 1)$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

#1: $\boxed{\theta = 0}$

#2: $\pi - 0 = \boxed{\pi}$

$2\cos\theta + 1 = 0$

$\cos\theta = -1/2$

$\theta = \cos^{-1}(-1/2)$

#1: $\boxed{\theta = \frac{2\pi}{3}}$

#2: $2\pi - \frac{2\pi}{3} = \boxed{\frac{4\pi}{3}}$

2) $2 = 3\cos\theta - \cos 2\theta$

$2 = 3\cos\theta - (2\cos^2\theta - 1)$

$2 = 3\cos\theta - 2\cos^2\theta + 1$

$2\cos^2\theta - 3\cos\theta + 1 = 0$

$(2\cos\theta - 1)(\cos\theta - 1) = 0$

$2\cos\theta - 1 = 0$

$\cos\theta = 1/2$

$\theta = \cos^{-1}(1/2)$

#1: $\boxed{\theta = \pi/3}$

#2: $2\pi - \pi/3 = \boxed{\frac{5\pi}{3}}$

$\cos\theta - 1 = 0$

$\cos\theta = 1$

$\theta = \cos^{-1}(1)$

#1: $\boxed{\theta = 0}$

#2: $2\pi - 0 = \cancel{2\pi}$

★ 3) $-\sin^2 2\theta + 2\sin^2\theta = 0$

$-4\sin^2\theta\cos^2\theta + 2\sin^2\theta = 0$

$2\sin^2\theta(-2\cos^2\theta + 1) = 0$

$2\sin^2\theta = 0$

$\sin^2\theta = 0$

$\sin\theta = \pm\sqrt{0}$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

#1: $\boxed{\theta = 0}$

#2: $\pi - 0 = \boxed{\pi}$

$-2\cos^2\theta + 1 = 0$

$-2\cos^2\theta = -1$

$\cos^2\theta = 1/2$

$\cos\theta = \pm\sqrt{1/2}$

$\cos\theta = \pm\sqrt{2}/2$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

#1: $\boxed{\theta = \pi/4}$

#2: $2\pi - \frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$

$\cos\theta = -\frac{\sqrt{2}}{2}$

$\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

#1: $\boxed{\theta = \frac{3\pi}{4}}$

#2: $2\pi - \frac{3\pi}{4} = \boxed{\frac{5\pi}{4}}$

Verify each identity.

4) $\sec^2 x - 2\cos^2 x = \tan^2 x - \cos 2x$ GIVEN

$\sec^2 x - (\cos 2x + 1) = \tan^2 x - \cos 2x$ DOUBLE-ANGLE ID.

$\sec^2 x - \cos 2x - 1 = \tan^2 x - \cos 2x$ DISTRIBUTE

$\tan^2 x - \cos 2x = \tan^2 x - \cos 2x$ ■ PYTHAGOREAN ID.

5) $\dfrac{1}{1 + \cos 2x} = \dfrac{\sec^2 x}{2}$ GIVEN

$\dfrac{1}{1 + 2\cos^2 x - 1} = \dfrac{\sec^2 x}{2}$ DOUBLE-ANGLE ID.

$\dfrac{1}{2\cos^2 x} = \dfrac{\sec^2 x}{2}$ ADD | SUBTRACT

$\dfrac{\sec^2 x}{2} = \dfrac{\sec^2 x}{2}$ ■ RECIPROCAL ID.

★ 6) $\dfrac{\cos 4x + 1 - \cos 2x}{\cos 2x} = 2\cos 2x - 1$ GIVEN

$\dfrac{\cos 2(2x) + 1 - \cos 2x}{\cos 2x} = 2\cos 2x - 1$ FACTOR | DIVIDE

$\dfrac{2\cos^2(2x) - 1 + 1 - \cos 2x}{\cos 2x} = 2\cos 2x - 1$ DOUBLE-ANGLE ID.

$\dfrac{2\cos^2(2x) - \cos 2x}{\cos 2x} = 2\cos 2x - 1$ ADD | SUBTRACT

$2\cos 2x - 1 = 2\cos 2x - 1$ ■ REDUCE | DIVIDE