Warm-Up

Verify the identity.

$$1 + \cos 2x + \sin^2 x + \tan^2 x = \cos^2 x + \sec^2 x$$

Verify the identity.

$1 + \cos 2x + \sin^2 x + \tan^2 x = \cos^2 x + \sec^2 x$	Given
$1 + (\cos^2 x - \sin^2 x) + \sin^2 x + \tan^2 x = \cos^2 x + \sec^2 x$	Double-Angle Identity
$1 + \cos^2 x + \tan^2 x = \cos^2 x + \sec^2 x$	Addition
$\cos^2 x + \sec^2 x = \cos^2 x + \sec^2 x$ ∎	Pythagorean Identity

10-4
More Proofs & More Solving Trigonometric Equations

Learning Targets:

- I can use identities to solve trigonometric equations

More Proofs!!!

Reminder:

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Proof Examples:

1) Verify: $$\frac{2\cos x}{1+\cos 2x} = \frac{1}{\cos x}$$

Given

$$\frac{2\cos x}{1+(2\cos^2 x - 1)} = \frac{1}{\cos x}$$

Double-Angle Identity

$$\frac{2\cos x}{2\cos^2 x} = \frac{1}{\cos x}$$

Addition

$$\frac{1}{\cos x} = \frac{1}{\cos x} \quad \blacksquare$$

Division

Proof Examples:

2) Verify: $2\sin x\,(\cos x - \sin x) = \sin 2x - 1 + \cos 2x$ Given

$$2\sin x\cos x - 2\sin^2 x = \sin 2x - 1 + \cos 2x$$ Distribute

$$\sin 2x - 2\sin^2 x = \sin 2x - 1 + \cos 2x$$ Double-Angle Identity

$$\sin 2x + \cos 2x - 1 = \sin 2x - 1 + \cos 2x \ \blacksquare$$ Double-Angle Identity

Proof Examples:

3) Verify: $\dfrac{1-\cos 2x}{\cos^2 x} = \dfrac{2}{\cot^2 x}$ Given

$$\frac{1-(1-2\sin^2 x)}{\cos^2 x} = \frac{2}{\cot^2 x}$$ Double-Angle Identity

$$\frac{2\sin^2 x}{\cos^2 x} = \frac{2}{\cot^2 x}$$ Distribute

$$2\tan^2 x = \frac{2}{\cot^2 x}$$ Quotient Identity

$$\frac{2}{\cot^2 x} = \frac{2}{\cot^2 x} \quad \blacksquare$$ Reciprocal Identity

Proof Examples:

4) Verify: $\dfrac{1}{\tan x\,(1-\cos 2x)} = \dfrac{\cot x}{2\sin^2 x}$ Given

$$\frac{1}{\tan x\left(1-(1-2\sin^2 x)\right)} = \frac{\cot x}{2\sin^2 x}$$ Double-Angle Identity

$$\frac{1}{\tan x\,(2\sin^2 x)} = \frac{\cot x}{2\sin^2 x}$$ Distribute

$$\frac{\cot x}{2\sin^2 x} = \frac{\cot x}{2\sin^2 x} \quad \blacksquare$$ Reciprocal Identity

More Solving!!!

Solving Examples:

1) Solve: $2\sin 2x = 1$ for $0° \leq x \leq 360°$

$$2\sin 2x = 1$$

Period = 180°

$$\sin 2x = \frac{1}{2}$$

$$2x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$2x = 30°$$

$$x = 15°, 195°$$

$$x = 75°, 255°$$

Solving Examples:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

2) Solve: $\cos 2x = 1 - \sin x$ for $0 \le x \le 2\pi$

$$\cos 2x = 1 - \sin x$$

$$1 - 2\sin^2 x = 1 - \sin x$$

$$-2\sin^2 x + \sin x = 0$$

$$\sin x(-2\sin x + 1) = 0$$

$\sin x = 0$ Period = 2π $-2\sin x + 1 = 0$

$x = 0, \pi$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \ \frac{5\pi}{6}$$

Solving Examples:

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

3) Solve: $3\cos 2x + \cos x = 2$ for $0 \le x \le 2\pi$

$$3\cos 2x + \cos x = 2$$
$$3(2\cos^2 x - 1) + \cos x = 2$$
$$6\cos^2 x - 3 + \cos x = 2$$
$$6\cos^2 x + \cos x - 5 = 0$$
$$(6\cos x - 5)(\cos x + 1) = 0$$

$$6\cos x - 5 = 0$$
$$\cos x = \frac{5}{6}$$
$$x \approx 0.59,\ 5.70$$

Period = 2π

$$\cos x + 1 = 0$$
$$\cos x = -1$$
$$x = \pi$$

You try: Solve for $0 \leq x \leq 2\pi$

4) $6\sin^2\theta - 4 = -\cos 2\theta$

5) $-3\sin 2\theta = -2\cos\theta - 2\sin\theta$

You try: Solve for $0 \leq x \leq 2\pi$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

4) $6\sin^2\theta - 4 = -\cos 2\theta$

$$6\sin^2\theta - 4 = -(1 - 2\sin^2\theta)$$

$$6\sin^2\theta - 4 = -1 + 2\sin^2\theta$$

$$4\sin^2\theta - 3 = 0$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \pm\sqrt{\frac{3}{4}}$$

$$\sin\theta = \pm\frac{\sqrt{3}}{2}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = -\frac{\pi}{3} \Rightarrow \theta = \frac{5\pi}{3}$$

$$\theta = \frac{4\pi}{3}$$

You try: Solve for $0 \leq x \leq 2\pi$

$\sin 2\theta = 2\sin\theta\cos\theta$

5) $\sin 2\theta = \sqrt{3}\cos\theta$

$$2\sin\theta\cos\theta = \sqrt{3}\cos\theta$$

$$2\sin\theta\cos\theta - \sqrt{3}\cos\theta = 0$$

$$\cos\theta\left(2\sin\theta - \sqrt{3}\right) = 0$$

$$\cos\theta = 0 \qquad\qquad 2\sin\theta - \sqrt{3} = 0$$

$$\theta = \frac{\pi}{2} \qquad\qquad \sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{3\pi}{2} \qquad\qquad \theta = \frac{\pi}{3} \qquad \theta = \frac{2\pi}{3}$$