

Proofs With Double-Angles

Verify each identity.

1) $\sin 2x + 1 - \cos 2x = 2\sin x \cdot (\sin x + \cos x)$

$$2\sin x \cos x + 1 - \cos 2x = 2\sin x (\sin x + \cos x)$$

$$2\sin x \cos x + 1 - (1 - 2\sin^2 x) = 2\sin x (\sin x + \cos x)$$

$$2\sin x \cos x + 1 - 1 + 2\sin^2 x = 2\sin x (\sin x + \cos x)$$

$$2\sin x \cos x + 2\sin^2 x = 2\sin x (\sin x + \cos x)$$

$$2\sin x (\cos x + \sin x) = 2\sin x (\sin x + \cos x) \quad \blacksquare$$

GIVEN

DOUBLE-ANGLE ID.

DOUBLE-ANGLE ID.

DISTRIBUTE

ADD/SUBTRACT

FACTOR (GCF)

2) $\frac{1}{\csc^2 x} = \cos^2 x - \cos 2x$

GIVEN

$$\sin^2 x = \cos^2 x - \cos 2x$$

RECIPROCAL ID.

$$-\cos 2x + \cos^2 x = \cos^2 x - \cos 2x$$

DOUBLE-ANGLE ID.

3) $\frac{2\cos^2 x}{\csc^2 x} = \sin^2 x (1 + \cos 2x)$

GIVEN

$$\sin^2 x \cdot 2\cos^2 x = \sin^2 x (1 + \cos 2x)$$

RECIPROCAL ID.

$$\sin^2 x (\cos 2x + 1) = \sin^2 x (1 + \cos 2x) \quad \blacksquare$$

DOUBLE-ANGLE ID.

$$4) \frac{1 + \cos 2x}{\sin^2 x} = \frac{2}{\tan^2 x}$$

GIVEN

$$\frac{1 + 2\cos^2 x - 1}{\sin^2 x} = \frac{2}{\tan^2 x}$$

DOUBLE-ANGLE ID.

$$\frac{2\cos^2 x}{\sin^2 x} = \frac{2}{\tan^2 x}$$

ADD/SUBTRACT

$$2\cot^2 x = \frac{2}{\tan^2 x}$$

QUOTIENT ID.

$$\frac{2}{\tan^2 x} = \frac{2}{\tan^2 x} \quad \blacksquare$$

RECIPROCAL ID.

$$5) \frac{\sin 2x}{\csc^2 x} = 2\sin^3 x \cos x$$

GIVEN

$$\frac{2\sin x \cos x}{\csc^2 x} = 2\sin^3 x \cos x$$

DOUBLE-ANGLE ID.

$$\sin^2 x \cdot 2\sin x \cos x = 2\sin^3 x \cos x$$

RECIPROCAL ID.

$$2\sin^3 x \cos x = 2\sin^3 x \cos x \quad \blacksquare$$

MULTIPLY.

$$6) \frac{2\sin x \cos x}{\cot x} = \tan x \sin 2x$$

GIVEN

$$\tan x \cdot 2\sin x \cos x = \tan x \sin 2x$$

RECIPROCAL ID.

$$\tan x \cdot \sin 2x = \tan x \sin 2x \quad \blacksquare$$

DOUBLE-ANGLE ID.

$$7) \frac{\sin 4x}{\sin 2x} = 2\cos 2x$$

GIVEN

$$\frac{\sin 4x}{2\sin x \cos x} = 2\cos 2x$$

DOUBLE-ANGLE ID.

$$\frac{\sin(2(2x))}{2\sin x \cos x} = 2\cos 2x$$

FACTOR / DIVIDE

$$\frac{2\sin 2x \cos 2x}{2\sin x \cos x} = 2\cos 2x$$

DOUBLE-ANGLE ID.

$$\frac{\sin 2x \cos 2x}{\sin x \cos x} = 2\cos 2x$$

DIVIDE / REDUCE

$$\frac{2\sin x \cos x \cdot \cos 2x}{\sin x \cos x} = 2\cos 2x$$

DOUBLE-ANGLE ID.

$$2\cos 2x = 2\cos 2x \quad \blacksquare$$

DIVIDE / REDUCE