

Double & Half-Angle Identities

Use the double and half-angle identities to find the exact value of each.

1) $2\cos^2 \frac{\pi}{8} - 1$

$$= \cos 2\left(\frac{\pi}{8}\right)$$

$$= \cos \frac{2\pi}{8}$$

$$= \cos \pi/4$$

$$= \boxed{\frac{\sqrt{2}}{2}}$$

2) $\frac{2\tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$

$$= \tan 2\left(\frac{\pi}{8}\right)$$

$$= \tan \frac{2\pi}{8}$$

$$= \tan \frac{\pi}{4}$$

$$= \boxed{1}$$

3) $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$

$$= \cos 2\left(\frac{\pi}{12}\right)$$

$$= \cos \frac{2\pi}{12}$$

$$= \cos \pi/6$$

$$= \boxed{\frac{\sqrt{3}}{2}}$$

4) $1 - 2\sin^2 \frac{7\pi}{12}$

$$= \cos 2\left(\frac{7\pi}{12}\right)$$

$$= \cos \frac{14\pi}{12}$$

$$= \cos \frac{7\pi}{6}$$

$$= \boxed{-\frac{\sqrt{3}}{2}}$$

$$5) 4\sin \frac{2\pi}{3} \cos \frac{2\pi}{3}$$

$$= 2 \left(2\sin \frac{2\pi}{3} \cos \frac{2\pi}{3} \right)$$

$$= 2 \left(\sin 2 \left(\frac{2\pi}{3} \right) \right)$$

$$= 2 \left(\sin \frac{4\pi}{3} \right)$$

$$= 2 \left(-\frac{\sqrt{3}}{2} \right)$$

$$= \boxed{-\sqrt{3}}$$

$$6) \sin 337.5^\circ$$

$$337.5 = \frac{675}{2}$$

$$\sin 337.5^\circ = \sin \frac{675^\circ}{2} = -\sqrt{\frac{1 - \cos 675^\circ}{2}}$$

$$= -\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2 - \sqrt{2}}{2}} = -\sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= \boxed{-\frac{\sqrt{2 - \sqrt{2}}}{2}}$$

$$7) \cos 285^\circ \quad 285 = \frac{570}{2}$$

$$\cos 285 = \cos \frac{570}{2} = \sqrt{\frac{1 + \cos 570}{2}}$$

$$= \sqrt{\frac{1 + \frac{-\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}}$$

$$= \boxed{\frac{\sqrt{2 - \sqrt{3}}}{2}}$$

$$8) \tan 22.5^\circ \quad 22.5 = \frac{45}{2}$$

$$\tan 22.5 = \tan \frac{45}{2} = \frac{1 - \cos 45}{\sin 45}$$

$$= \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}}$$

$$= \frac{2 - \sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}} = \frac{4 - 2\sqrt{2}}{2\sqrt{2}}$$

$$= \frac{2 - \sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2} - 2}{2}$$

$$= \frac{\sqrt{2} - 1}{1} = \boxed{\sqrt{2} - 1}$$