Warm-Up

Verify the identity.

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$\dfrac{\sec^2 x - 1}{\sec x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\tan^2 x}{\sec x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{\sin x}{\cos x}\right)^2}{\dfrac{1}{\cos x}}$ Simplify

$\dfrac{\sin^2 x}{\cos x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\tan x \sin x$ ■

Unit 3

10-3 Notes:
Double-Angle and Half-Angle Formulas

Learning Targets:

- I can apply double-angle and half-angle formulas

Double-Angle Formulas

In the double-angle formulas, the θ for each trig function is doubled.

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

Examples: Simplify the expression.

1) $2\sin 10° \cos 10°$

$$2\sin 10° \cos 10° = \sin 2(10°) = \sin 20°$$

Examples: Simplify the expression.

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

2) $\sin^2\left(\frac{\alpha}{2}\right) - \cos^2\left(\frac{\alpha}{2}\right)$

$$\sin^2\left(\frac{\alpha}{2}\right) - \cos^2\left(\frac{\alpha}{2}\right) = -1\left(\cos^2\left(\frac{\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)\right)$$

$$= -1\left(\cos 2\left(\frac{\alpha}{2}\right)\right)$$

$$= -\cos\alpha$$

Examples: Simplify the expression.

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

3) $\dfrac{2\tan 157.5°}{1-\tan^2 157.5°}$

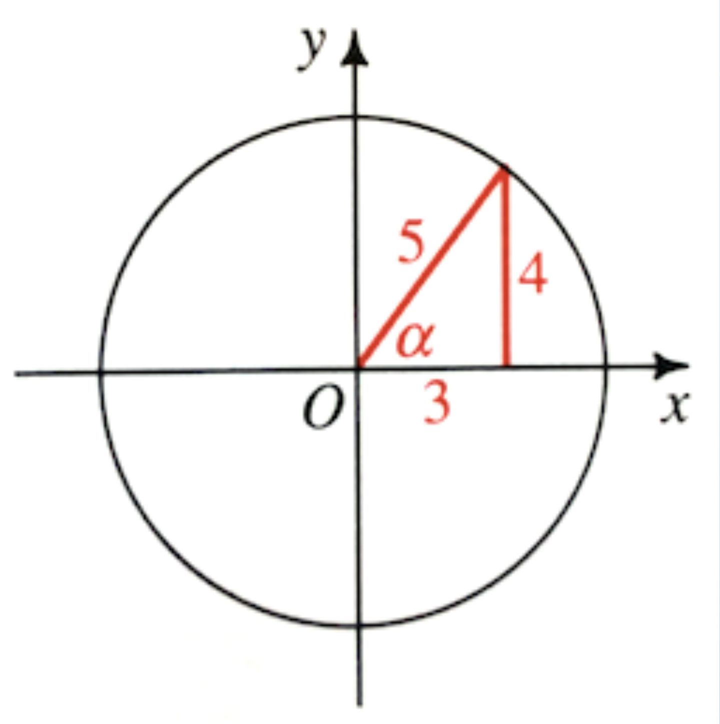
$$\frac{2\tan 157.5°}{1-\tan^2 157.5°} = \tan 2(157.5°) = \tan 315° = -1$$

Examples:

4) If $\sin\alpha = \frac{4}{5}$ and $0 \le \alpha \le \frac{\pi}{2}$, find $\sin 2\alpha$, $\cos 2\alpha$, and $\tan 2\alpha$.

$$\sin\alpha = \frac{4}{5} = \frac{opp}{hyp}$$

We can use the Pythagorean Theorem to find the missing side.



$$\cos\alpha = \frac{3}{5}$$

$$\tan\alpha = \frac{4}{3}$$

Examples:

4) If $\sin\alpha = \frac{4}{5}$ and $0 \le \alpha \le \frac{\pi}{2}$, find $\sin 2\alpha$, $\cos 2\alpha$, and $\tan 2\alpha$.

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

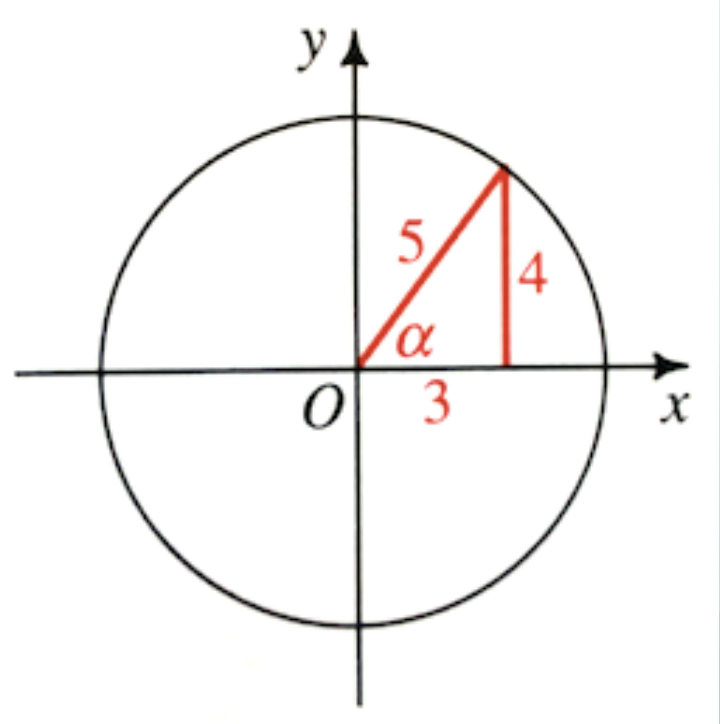
$$= \frac{24}{25}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$$

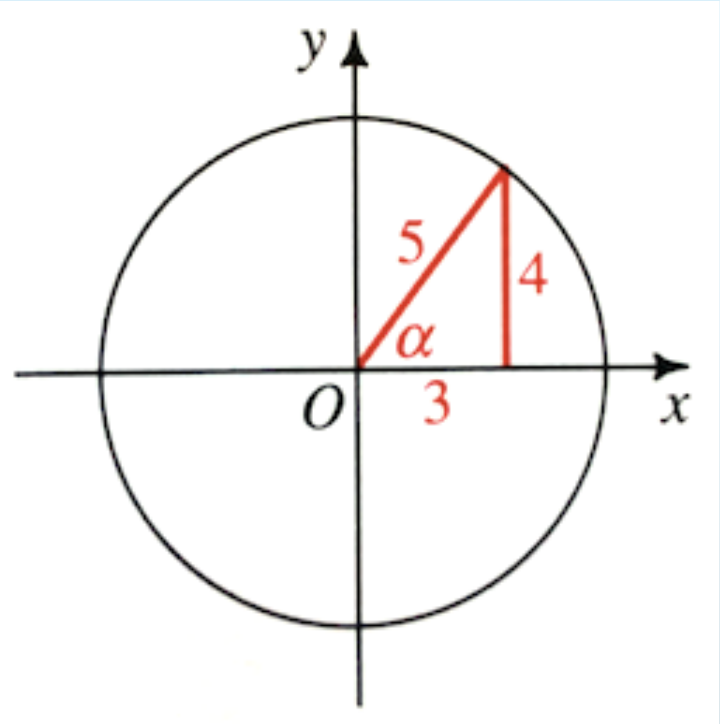
$$= \frac{9}{25} - \frac{16}{25}$$

$$= -\frac{7}{25}$$



Examples:

4) If $\sin\alpha = \frac{4}{5}$ and $0 \le \alpha \le \frac{\pi}{2}$, find $\sin 2\alpha$, $\cos 2\alpha$, and $\tan 2\alpha$.



$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$= \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{1-\frac{16}{9}} = \frac{\frac{8}{3}}{\frac{9}{9}-\frac{16}{9}}$$

$$= \frac{\frac{8}{3}}{-\frac{7}{9}} = \frac{8}{3}\times-\frac{9}{7} = -\frac{24}{7}$$

Half-Angle Formulas

When you use the half-angle formulas, choose + or – depending on the quadrant in which $\frac{\theta}{2}$ lies.

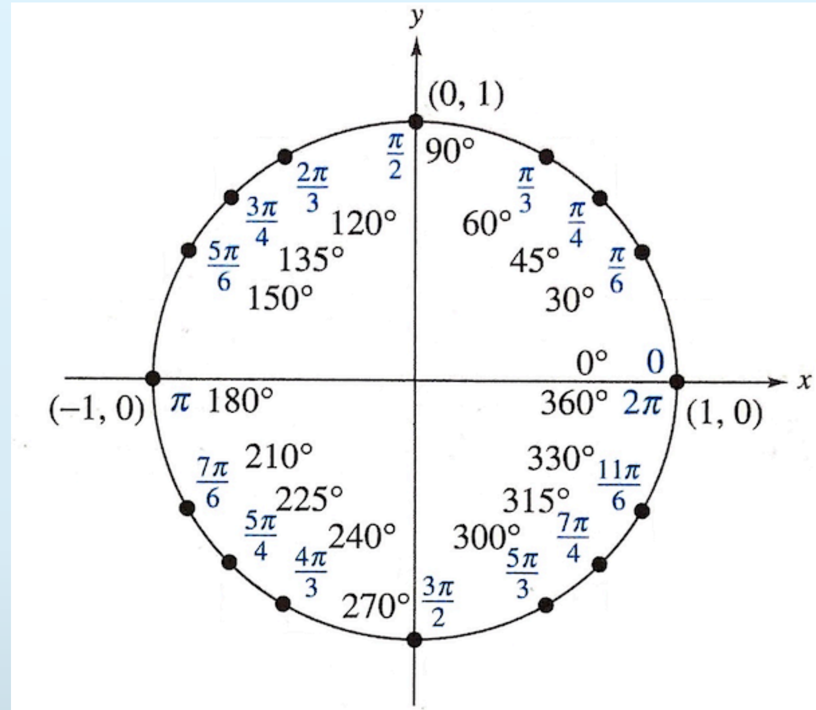
$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

Examples:

1) Find the exact value of $\cos\frac{5\pi}{8}$.



In which quadrant is $\frac{5\pi}{8}$?

2nd quadrant, so $\cos\frac{5\pi}{8}$ is negative.

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

Examples:

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

1) Find the exact value of $\cos\frac{5\pi}{8}$.

Since $\frac{5\pi}{8} = \frac{1}{2}\left(\frac{5\pi}{4}\right)$, we can let $x = \frac{5\pi}{4}$. So,

$$\cos\frac{\frac{5\pi}{4}}{2} = -\sqrt{\frac{1+\cos\left(\frac{5\pi}{4}\right)}{2}}$$

$$\cos\frac{5\pi}{8} = -\sqrt{\frac{1+-\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2}{2}+-\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{2}}{4}} = -\frac{\sqrt{2-\sqrt{2}}}{2}$$

Examples: Simplify the expression.

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

2) $\sin(-67.5°)$

$$\sin(-67.5°) = \sin\left(-\frac{135°}{2}\right) = -\sqrt{\frac{1-\cos 135°}{2}} = -\sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= -\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = -\frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}} = -\frac{\sqrt{2+\sqrt{2}}}{2} = -\frac{1}{2}\sqrt{2+\sqrt{2}}$$

<u>Examples</u>: Simplify the expression.

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

3) $\cos\left(\frac{\pi}{8}\right)$

$$\cos\left(\frac{\pi}{8}\right) = \cos\left(\frac{\frac{\pi}{4}}{2}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}} = \frac{\sqrt{2+\sqrt{2}}}{2} = \frac{1}{2}\sqrt{2+\sqrt{2}}$$

Examples: Simplify the expression.

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

4) $\tan\left(\frac{11\pi}{12}\right)$

$$\tan\left(\frac{11\pi}{12}\right) = \tan\left(\frac{\frac{11\pi}{6}}{2}\right) = \frac{1-\cos\frac{11\pi}{6}}{\sin\frac{11\pi}{6}}$$

$$= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\left(1-\frac{\sqrt{3}}{2}\right) = 2-\sqrt{3}$$