7-6 Notes

The Inverse Trig Functions

The inverse trig functions are denoted:

$\sin^{-1}(\theta) = \arcsin(\theta)$

$\cos^{-1}(\theta) = \arccos(\theta)$

$\tan^{-1}(\theta) = \arctan(\theta)$

$\csc^{-1}(\theta) = \operatorname{arccsc}(\theta) = \sin^{-1}\frac{1}{(\theta)}$

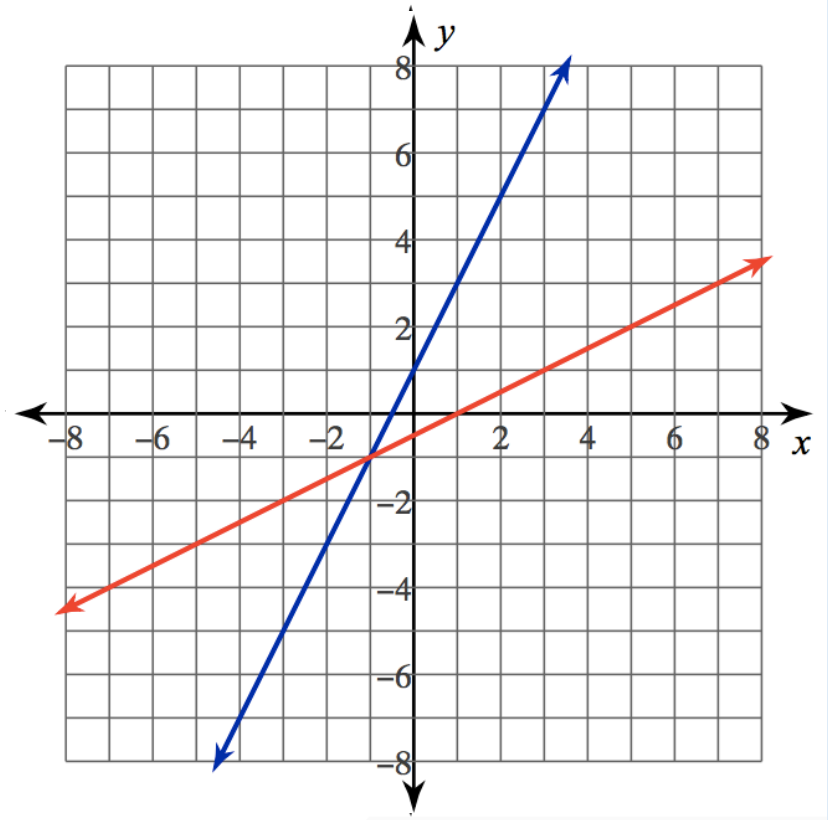
$\sec^{-1}(\theta) = \operatorname{arcsec}(\theta) = \cos^{-1}\frac{1}{(\theta)}$

$\cot^{-1}(\theta) = \operatorname{arccot}(\theta) = \tan^{-1}\frac{1}{(\theta)}$

Example: $\cos^{-1}\left(\frac{1}{2}\right) = \sec^{-1}(2)$

Graphs of inverses

Generally, to find the inverse of a function, you switch the *x* and *y* variables.



$f(x)$: $y = 2x + 1$

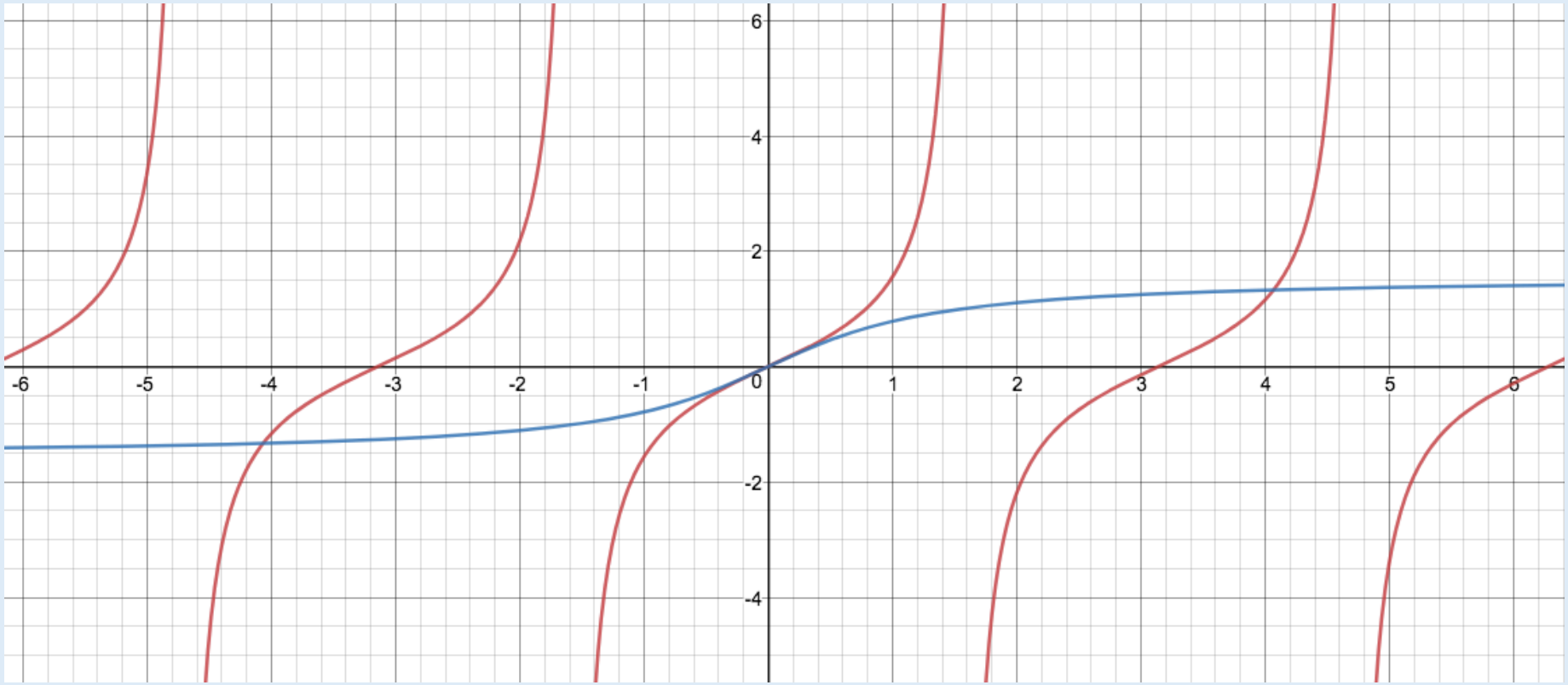
$f^{-1}(x)$: $x = 2y + 1$

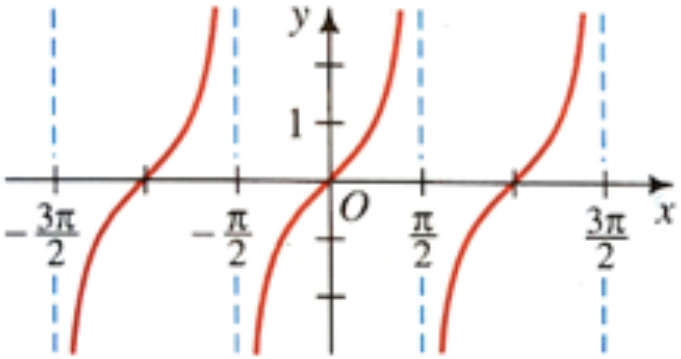
$x - 1 = 2y$

$\frac{x}{2} - \frac{1}{2} = y$

Inverses are always reflections of each other over the line $y = x$.

$\tan^{-1}(\theta)$

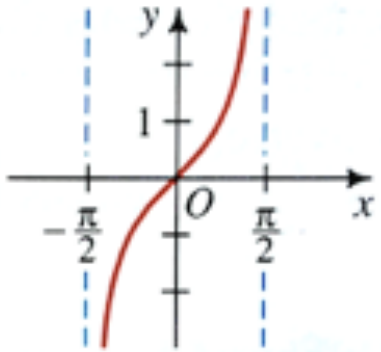




$f(x) = \tan x$ has no inverse.

Domain: $\{x \mid x \neq \frac{\pi}{2} + n\pi\}$

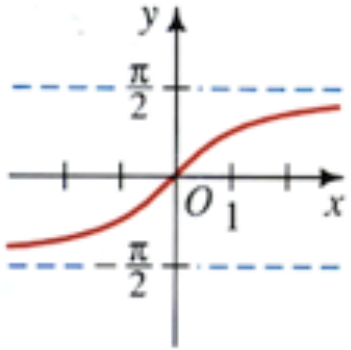
Range: Real numbers



$F(x) = \mathrm{Tan}\, x$ has an inverse.

Domain: $\{x \mid -\frac{\pi}{2} < x < \frac{\pi}{2}\}$

Range: Real numbers



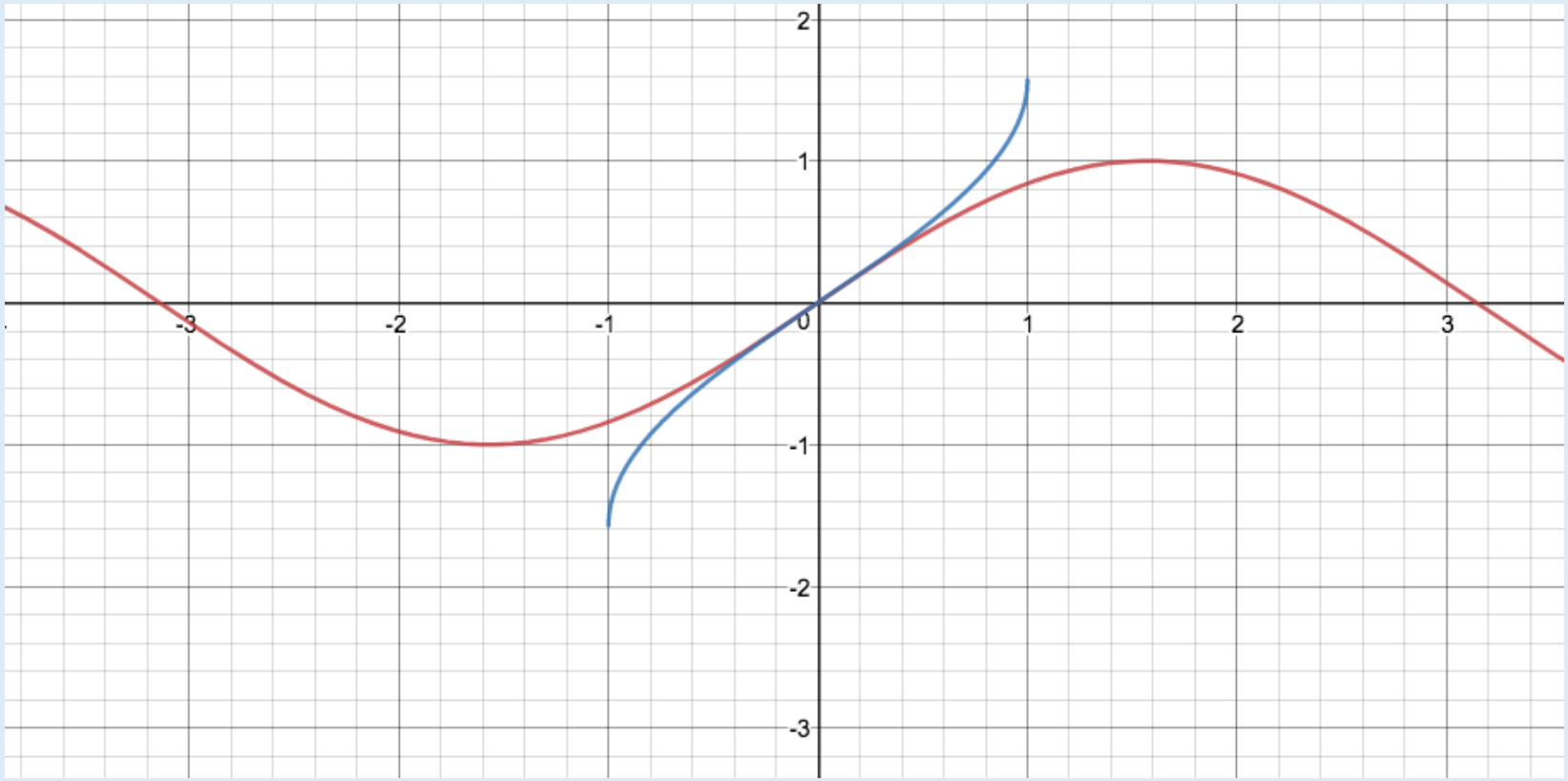
$F^{-1}(x) = \mathrm{Tan}^{-1} x$

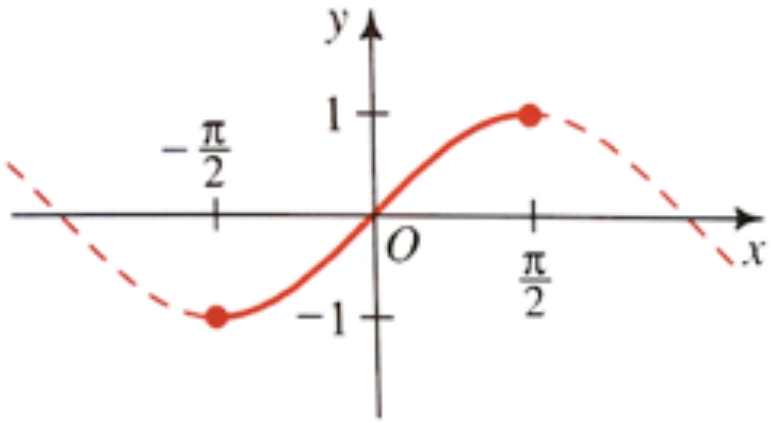
Domain: Real numbers

Range: $\{y \mid -\frac{\pi}{2} < y < \frac{\pi}{2}\}$

quadrants 4 & 1

$\sin^{-1}(\theta)$



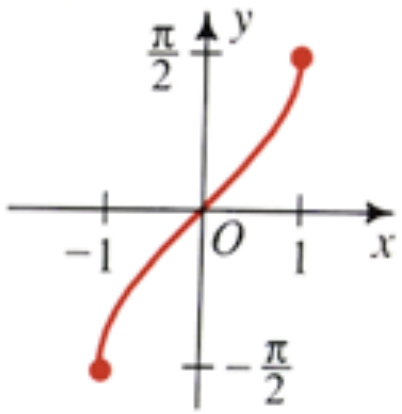


$G(x) = \text{Sin } x$ has an inverse.

Domain: $\{x \mid -\frac{\pi}{2} \le x \le \frac{\pi}{2}\}$

Range: $\{y \mid -1 \le y \le 1\}$

quadrants 4 & 1

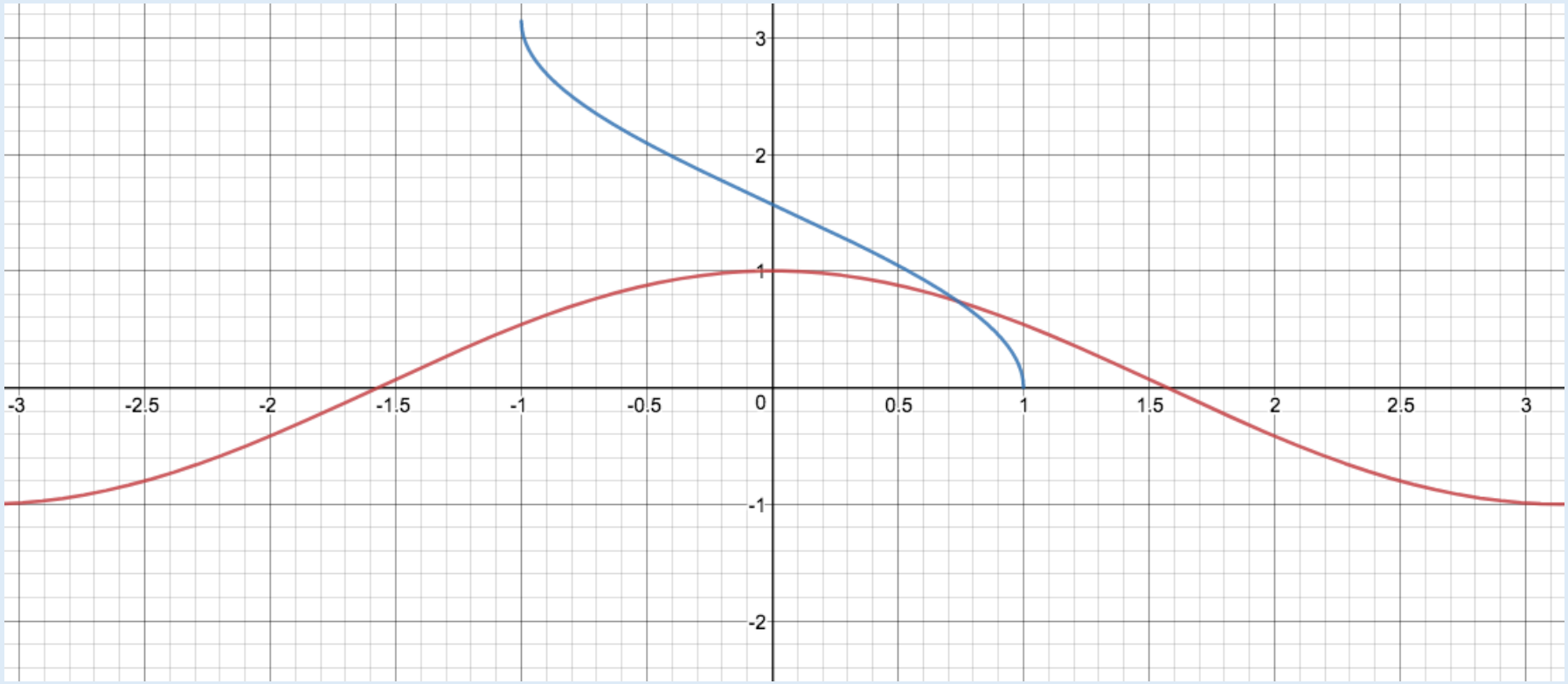


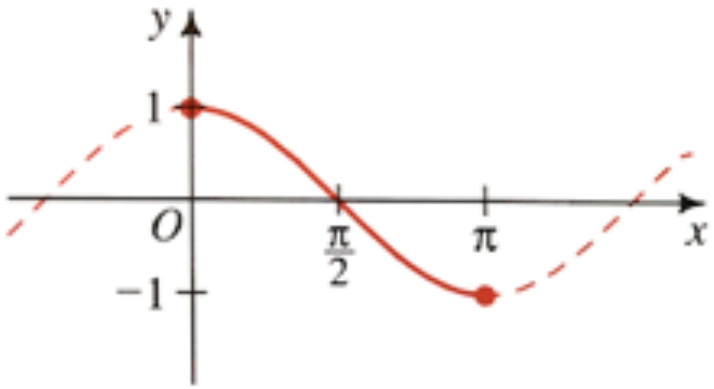
$G^{-1}(x) = \text{Sin}^{-1} x$

Domain: $\{x \mid -1 \le x \le 1\}$

Range: $\{y \mid -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$

$\cos^{-1}(\theta)$



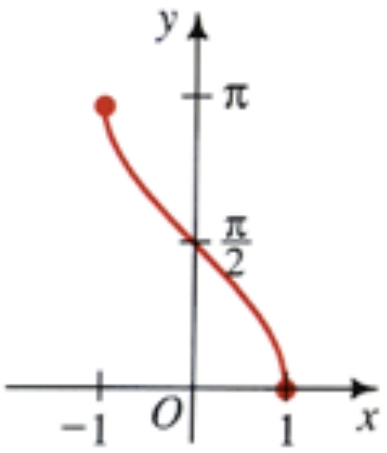


$H(x) = \text{Cos}\, x$ has an inverse.

Domain: $\{x \mid 0 \le x \le \pi\}$

Range: $\{y \mid -1 \le y \le 1\}$

quadrants 1 & 2



$H^{-1}(x) = \text{Cos}^{-1}\, x$

Domain: $\{x \mid -1 \le x \le 1\}$

Range: $\{y \mid 0 \le y \le \pi\}$

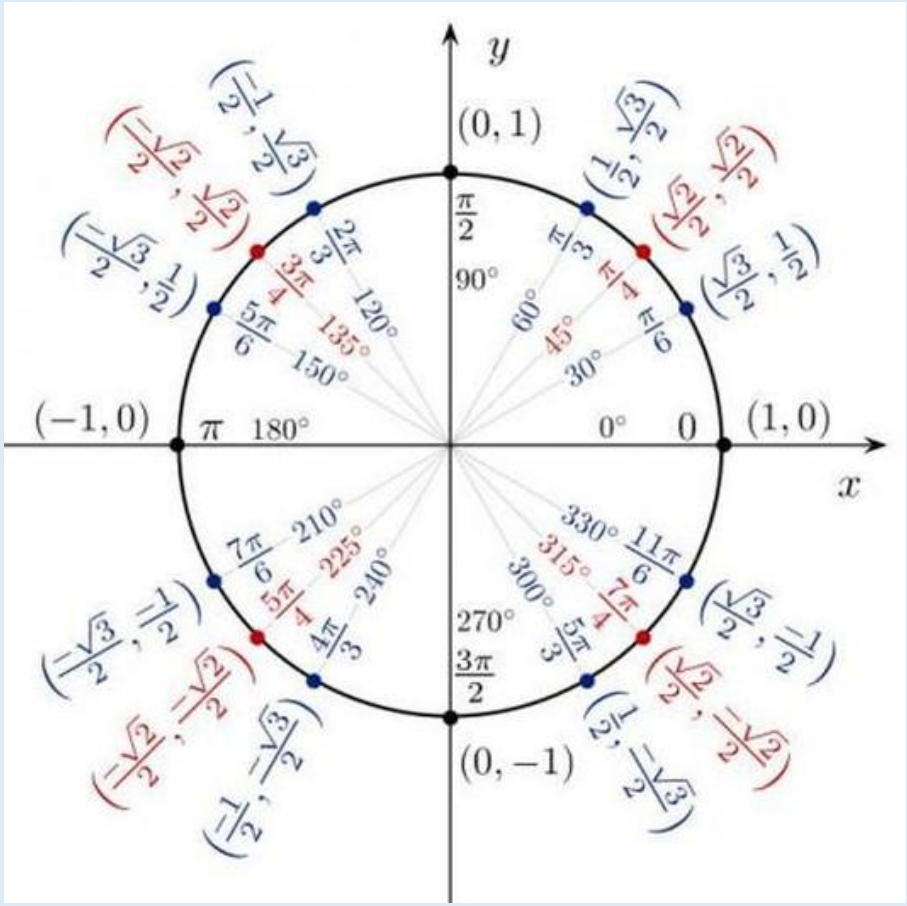
Using The Unit Circle

Where in the circle do you look to find the answer?

$\text{Tan}^{-1}(\theta)$ ⟶ Use quadrants 1 & 4

$\text{Sin}^{-1}(\theta)$ ⟶ Use quadrants 1 & 4

$\text{Cos}^{-1}(\theta)$ ⟶ Use quadrants 1 & 2



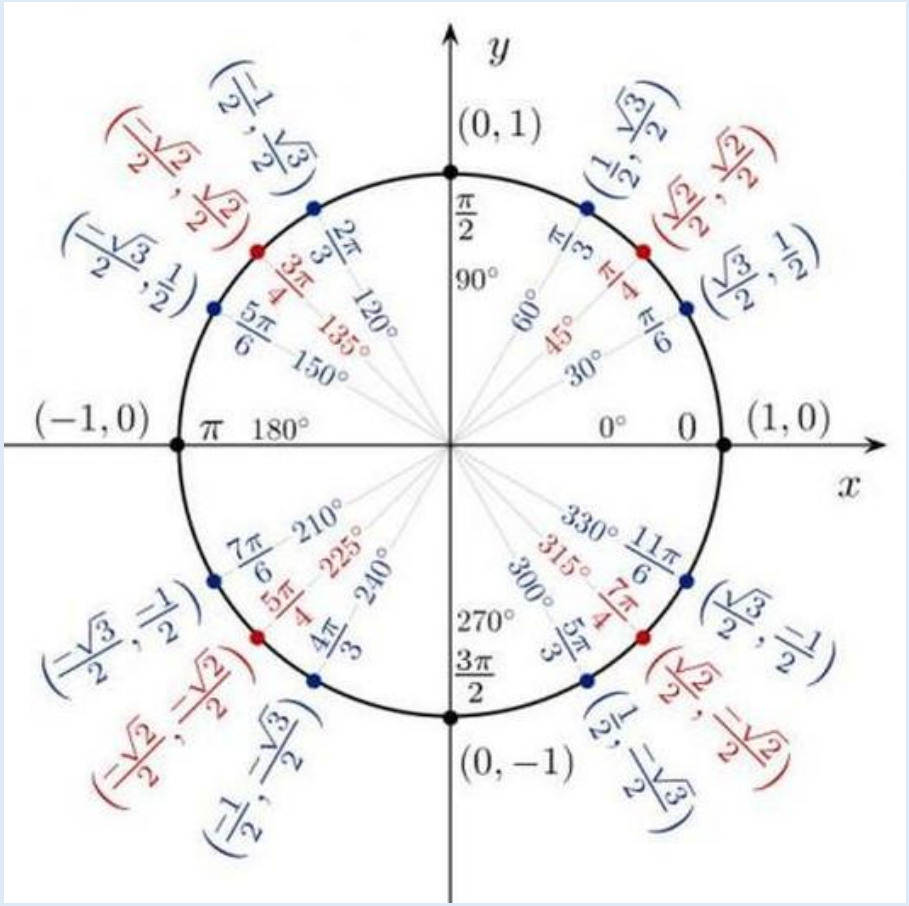
Using The Unit Circle

Without using a calculator, find the exact value of:

1) $\text{Tan}^{-1}(1) = \frac{\pi}{4}$

2) $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

3) $\text{Tan}^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$



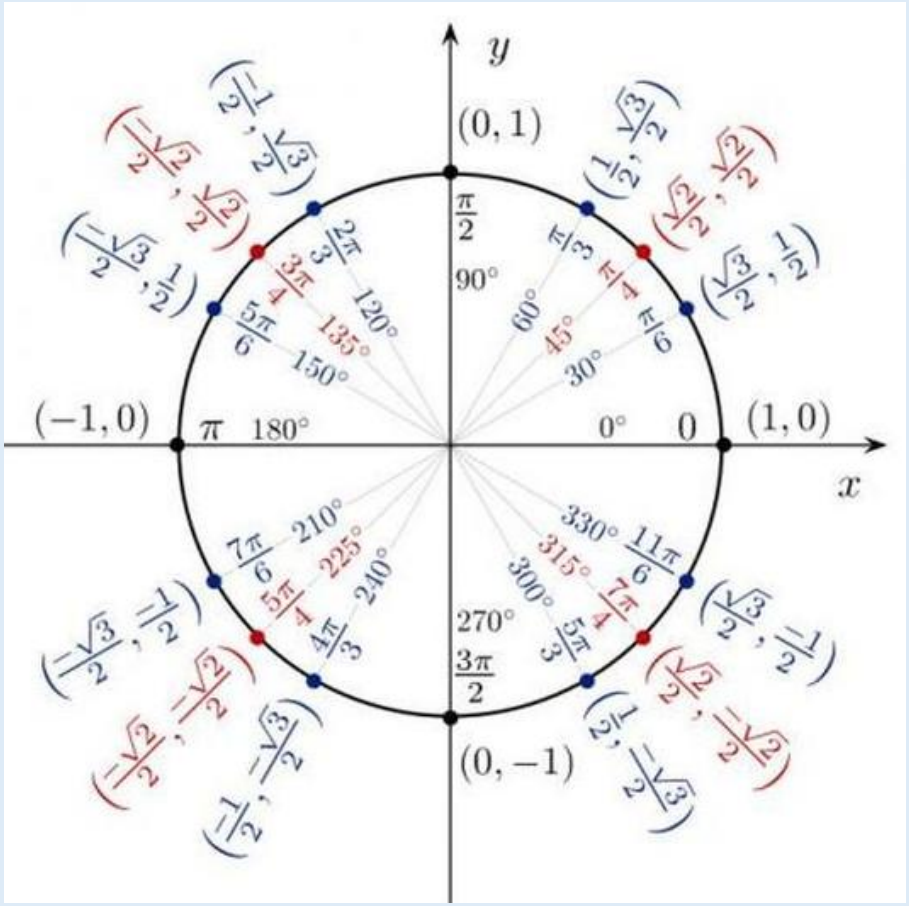
Using The Unit Circle

Without using a calculator, find the exact value of:

4) $\text{Sin}^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

5) $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

6) $\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$



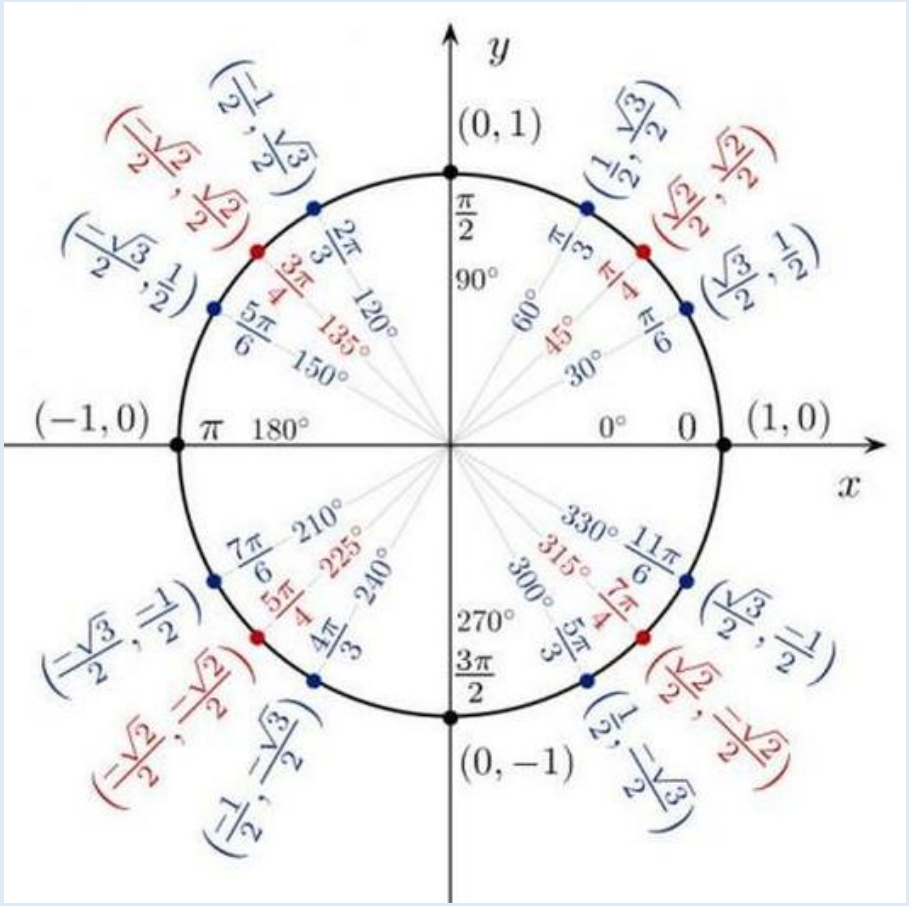
Using The Unit Circle

Without using a calculator, find the exact value of:

7) $\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

8) $\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

9) $\text{Cos}^{-1}(-1) = \pi$



Practice Problems

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Written exercises #3-7