

Product-to-Sum & Sum-to-Product

Write each product as a sum or difference with positive arguments.

1) $\cos 46^\circ \cos 58^\circ$

$$= \frac{1}{2} [\cos(46-58) + \cos(46+58)]$$

$$= \frac{1}{2} (\cos -12 + \cos 104)$$

2) $\sin 101^\circ \sin 3^\circ$

$$= \frac{1}{2} [\cos(101-3) - \cos(101+3)]$$

$$= \frac{1}{2} (\cos 98 - \cos 104)$$

3) $5 \cos 44^\circ \sin 19^\circ$

$$= 5 \cdot \frac{1}{2} [\sin(44+19) - \sin(44-19)]$$

$$= \frac{5}{2} (\sin 63 - \sin 25)$$

4) $\sin 101^\circ \cos 3^\circ$

$$= \frac{1}{2} [\sin(101+3) + \sin(101-3)]$$

$$= \frac{1}{2} (\sin 104 + \sin 98)$$

5) $\sin 7\theta \cos 2\theta$

$$= \frac{1}{2} [\sin(7\theta+2\theta) + \sin(7\theta-2\theta)]$$

$$= \frac{1}{2} (\sin 9\theta + \sin 5\theta)$$

6) $\sin 2A \sin 9A$

$$= \frac{1}{2} [\cos(2A-9A) - \cos(2A+9A)]$$

$$= \frac{1}{2} (\cos -7A - \cos 11A)$$

7) $\sin 3\theta \cos \theta$

$$= \frac{1}{2} [\sin(3\theta+\theta) + \sin(3\theta-\theta)]$$

$$= \frac{1}{2} (\sin 4\theta + \sin 2\theta)$$

8) $-5 \cos 4B \sin 7B$

$$= -5 \cdot \frac{1}{2} [\sin(4B+7B) - \sin(4B-7B)]$$

$$= \frac{-5}{2} (\sin 11B - \sin -3B)$$

Write each sum or difference as a product with positive arguments.

9) $\sin 12^\circ + \sin 30^\circ$

$$= 2 \sin\left(\frac{12+30}{2}\right) \cos\left(\frac{12-30}{2}\right)$$

$$= 2 \sin\left(\frac{42}{2}\right) \cos\left(\frac{-18}{2}\right)$$

$$= \boxed{2 \sin 21 \cos -9}$$

10) $3\cos 55^\circ + 3\cos 231^\circ$

$$= 3 \cdot 2 \cos\left(\frac{55+231}{2}\right) \cos\left(\frac{55-231}{2}\right)$$

$$= 6 \cos\left(\frac{286}{2}\right) \cos\left(\frac{-176}{2}\right)$$

$$= \boxed{6 \cos 143 \cos -88}$$

11) $\sin 263^\circ - \sin 21^\circ$

$$= 2 \cos\left(\frac{263+21}{2}\right) \sin\left(\frac{263-21}{2}\right)$$

$$= 2 \cos\left(\frac{284}{2}\right) \sin\left(\frac{242}{2}\right)$$

$$= \boxed{2 \cos 142 \sin 121}$$

12) $\cos 49^\circ - \cos 151^\circ$

$$= -2 \sin\left(\frac{49+151}{2}\right) \sin\left(\frac{49-151}{2}\right)$$

$$= -2 \sin\left(\frac{200}{2}\right) \sin\left(\frac{-102}{2}\right)$$

$$= \boxed{-2 \sin 100 \sin -51}$$

13) $\sin 12B + \sin 6B$

$$= 2 \sin\left(\frac{12B+6B}{2}\right) \cos\left(\frac{12B-6B}{2}\right)$$

$$= 2 \sin\left(\frac{18B}{2}\right) \cos\left(\frac{6B}{2}\right)$$

$$= \boxed{2 \sin 9B \cos 3B}$$

14) $-2\sin 14A - 2\sin 6A$

$$= -2 \cdot 2 \cos\left(\frac{14A+6A}{2}\right) \sin\left(\frac{14A-6A}{2}\right)$$

$$= -4 \cos\left(\frac{20A}{2}\right) \sin\left(\frac{8A}{2}\right)$$

$$= \boxed{-4 \cos 10A \sin 4A}$$

15) $\cos 8B + \cos 6B$

$$= 2 \cos\left(\frac{8B+6B}{2}\right) \cos\left(\frac{8B-6B}{2}\right)$$

$$= 2 \cos\left(\frac{14B}{2}\right) \cos\left(\frac{2B}{2}\right)$$

$$= \boxed{2 \cos 7B \cos B}$$

16) $\cos 3\theta - \cos 15\theta$

$$= -2 \sin\left(\frac{3\theta+15\theta}{2}\right) \sin\left(\frac{3\theta-15\theta}{2}\right)$$

$$= -2 \sin\left(\frac{18\theta}{2}\right) \sin\left(\frac{-12\theta}{2}\right)$$

$$= \boxed{-2 \sin 9\theta \sin -6\theta}$$