Unit 3

10-2 NOTES
FORMULAS FOR $\mathrm{TAN}(\alpha \pm \beta)$

Learning Targets:

- I can apply formulas for $\tan(\alpha \pm \beta)$

SUM AND DIFFERENCE FORMULAS

TANGENT:

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

EXAMPLES:

1) Find the exact value of $\tan 105°$.

$$\tan 105° = \tan(60° + 45°) = \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$$

$$= \frac{\left(\frac{\sqrt{3}/2}{1/2}\right) + \left(\frac{\sqrt{2}/2}{\sqrt{2}/2}\right)}{1 - \left(\frac{\sqrt{3}/2}{1/2}\right)\left(\frac{\sqrt{2}/2}{\sqrt{2}/2}\right)} = \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$$

We have to rationalize the denominator

EXAMPLES:

1) Find the exact value of $\tan 105°$.

$$\frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{\sqrt{3}+3+1+\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3} = \frac{4+2\sqrt{3}}{-2} = \frac{2+1\sqrt{3}}{-1} = -2-\sqrt{3}$$

EXAMPLES:

2) Find the exact value of $\tan\frac{5\pi}{12}$. $\left(\frac{2\pi}{3}-\frac{\pi}{4}=\frac{8\pi}{12}-\frac{3\pi}{12}=\frac{5\pi}{12}\right)$

$$\tan\frac{5\pi}{12}=\tan\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)=\frac{\tan\frac{2\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{2\pi}{3}\tan\frac{\pi}{4}}$$

We have to rationalize the denominator

$$=\frac{\left(\frac{\sqrt{3}/2}{-1/2}\right)-\left(\frac{\sqrt{2}/2}{\sqrt{2}/2}\right)}{1+\left(\frac{\sqrt{3}/2}{-1/2}\right)\left(\frac{\sqrt{2}/2}{\sqrt{2}/2}\right)}=\frac{-\sqrt{3}-1}{1+(-\sqrt{3})(1)}=\frac{-\sqrt{3}-1}{1-\sqrt{3}}$$

EXAMPLES:

2) Find the exact value of $\tan \frac{5\pi}{12}$.

$$\frac{-\sqrt{3}-1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{-\sqrt{3}-3-1-\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3} = \frac{-4-2\sqrt{3}}{-2} = 2+\sqrt{3}$$

EXAMPLES:

3) Find the exact value of $\dfrac{\tan 62° - \tan 17°}{1 + \tan 62° \tan 17°}$

$$\frac{\tan 62° - \tan 17°}{1 + \tan 62° \tan 17°} = \tan(62° - 17°) = \tan 45° = 1$$

EXAMPLES:

4) Find the exact value of $\dfrac{\tan\frac{11\pi}{18}+\tan\frac{\pi}{18}}{1-\tan\frac{11\pi}{18}\tan\frac{\pi}{18}}$

$$\frac{\tan\frac{11\pi}{18}+\tan\frac{\pi}{18}}{1-\tan\frac{11\pi}{18}\tan\frac{\pi}{18}} = \tan\left(\frac{11\pi}{18}+\frac{\pi}{18}\right) = \tan\frac{12\pi}{18} = \tan\frac{2\pi}{3} = -\sqrt{3}$$