

Warm-Up

Verify the identity.

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$\sin^2 x \sec^2 x$ Use $\sec x = \frac{1}{\cos x}$

$\frac{\sin^2 x}{\cos^2 x}$ Use $\tan x = \frac{\sin x}{\cos x}$

$\tan^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$\sec^2 x - 1$ ■

10-1 NOTES PART 1: (SUM & DIFFERENCE FORMULAS)

FORMULAS FOR $\cos(\alpha \pm \beta)$ AND $\sin(\alpha \pm \beta)$

Learning Targets:

- I can apply formulas for $\cos(\alpha \pm \beta)$ and $\sin(\alpha \pm \beta)$

THE SUM AND DIFFERENCE FORMULAS

If we want to find the exact trig value of an angle that isn't one of our major angles in the unit circle, we can use a sum/difference formula.

SUM AND DIFFERENCE FORMULAS

SINE:

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

Or...

$$\sin(a \pm b) = \cos a \cos b \pm \sin a \sin b$$

SUM AND DIFFERENCE FORMULAS

COSINE:

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

Or...

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

EXAMPLE 1: Find the exact value of $\cos 75^\circ$.

75° is not on the unit circle, but 30° and 45° are.

$$\begin{aligned}\cos 75^\circ &= \cos(30^\circ + 45^\circ) \\ &= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ \\ &= \left(\frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2} * \frac{\sqrt{2}}{2}\right) \\ &= \left(\frac{\sqrt{6}}{4}\right) - \left(\frac{\sqrt{2}}{4}\right) \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

EXAMPLE 2: Find the exact value of $\sin \frac{\pi}{12}$.

$\frac{\pi}{12}$ is not on the unit circle, but $\frac{\pi}{3}$ and $\frac{\pi}{4}$ are.

$$\frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$$

$$\sin \frac{\pi}{12} = \sin \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$$

$$= \left(\frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} \right) - \left(\frac{1}{2} * \frac{\sqrt{2}}{2} \right)$$

$$= \left(\frac{\sqrt{6}}{4} \right) - \left(\frac{\sqrt{2}}{4} \right)$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

EXAMPLE 3: Find the exact value of the expression:

$$\sin 42^\circ \cos 12^\circ - \cos 42^\circ \sin 12^\circ$$

This expression matches: $\sin(a - b) = \sin a \cos b - \cos a \sin b$

$$\sin(42^\circ - 12^\circ) = \sin 42^\circ \cos 12^\circ - \cos 42^\circ \sin 12^\circ$$

$$\sin(30^\circ) =$$

$$\frac{1}{2} =$$

Find the exact value of each.

1) $\cos 195$

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

2) $\sin -15$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

3) $\sin 38 \cos 82 + \cos 38 \sin 82$

$$\frac{\sqrt{3}}{2}$$

4) $\cos 163 \cos 17 - \sin 163 \sin 17$

$$-1$$