

heh

## Logs – Group Problems

\*\*1) Find the numerical value of  $N$  if  $N = (\log_6 24 - \log_6 12) * \log_8 36$

$$\begin{aligned}
 N &= (\log_6 \frac{24}{12}) * \log_8 36 \\
 &= \log_6 2 * \log_8 36 \\
 &= \log_6 2 * \frac{\log_6 36}{\log_6 8} \\
 &= \log_6 2 * \frac{2 \log_6 2}{\log_6 8} \\
 &\quad \leftarrow \text{cancel } \log_6 2 \\
 &= \frac{2 \log_6 2}{\log_6 8} \\
 &= \frac{\log_6 2^2}{\log_6 8} \\
 &= \frac{\log_6 4}{\log_6 8} \\
 &= \frac{\log_6 4}{\log_6 8} \\
 &= \boxed{\frac{2}{3}}
 \end{aligned}$$

answer:  $N = \frac{2}{3}$

\*2) Solve:  $\log_6(x+2) + \log_6(x-3) = 1$

$$\begin{aligned}
 \bullet \log_6 [(x+2)(x-3)] &= 1 \\
 \bullet \log_6 [x^2 - 3x + 2x - 6] &= 1 \\
 \bullet \log_6 (x^2 - 1x - 6) &= 1 \\
 6 &= x^2 - 1x - 6 \\
 0 &= x^2 - 1x - 12
 \end{aligned}$$

$\Rightarrow 0 = (x-4)(x+3)$

$\boxed{x=4}, \boxed{x=-3}$

answer:  $x = 4, -3$

\*\*3) Find  $\log_b \sqrt[3]{\frac{7}{8}}$  if  $\log_b 7 = .6263$  and  $\log_b 4 = .4462$ . Round your answer to 4 decimal

$$\begin{aligned}
 \log_b \left(\frac{7}{8}\right)^{1/3} &= \log_b 7^{1/3} - \log_b 2^{1/3} \\
 &= \frac{1}{3} \log_b 7 - \log_b 2^{1/2} \\
 &= \frac{1}{3} (.6263) - \frac{1}{2} (.4462) \\
 &= \boxed{-0.0143}
 \end{aligned}$$

answer: -0.0143

\*4) Find the exact value of  $x$ , if  $\log x$  is the average of  $\log 3$  and  $\log 16$ .

$$\log x = \frac{\log 3 + \log 16}{2}$$

$$= \log \sqrt{3} + \log 4$$

$$\log x = \boxed{\log 4\sqrt{3}}$$

answer:  $x = 4\sqrt{3}$

\*\*5) Solve for  $x$ :  $\log_3(x+3) - \log_3(x-5) = 2$

$$\begin{aligned}
 \log_3 \left( \frac{x+3}{x-5} \right) &= 2 \\
 3^2 &= \frac{x+3}{x-5} \\
 9 &= \frac{x+3}{x-5}
 \end{aligned}$$

answer:  $x = 6$

$$9(x-5) = x+3$$

$$9x - 45 = x+3$$

$$8x = 48$$

$$\boxed{x=6}$$

\*6) Find  $x$  if  $27^{\log_{27} 9} = 8x + 5$

$$\begin{aligned}\log_{27} 8x + 5 &= \log_{27} 9 \\ 8x + 5 &= 9 \\ x &= \frac{1}{2}\end{aligned}$$

answer:  $x = \frac{1}{2}$

\*\*7) Solve for  $x$ :  $\log_{10}(x^2 + 3x) + \log_{10} 5x = 1 + \log_{10} 2x$

$$\begin{aligned}\log(x^2 + 3x) + \log 5x - \log 2x &= 1 & 0 = 5x(x^2 + 3x - 1) \\ \log\left(\frac{(x^2 + 3x)5x}{2x}\right) &= 1 & \rightarrow 0 = 5x(x+4)(x-1) \\ 10 &= \frac{(x^2 + 3x)(5x)}{2x} & x=0, x=-4, x=1 \\ 20x &= 5x^3 + 15x^2 & \\ 0 &= 5x^3 + 15x^2 - 20x &\end{aligned}$$

answer:  $x = 0, -4, 1$

\*\*\*8) For what value(s) of  $x$ , where  $x$  is a real number, is  $\log_9 16 * \log_8 3 + \log_8 x = \log_8 3$

$$\begin{aligned}\log_9 16 * \log_8 3 &= \log_8 3 - \log_8 x & \log_8 9 + \log_8 2 \\ \log_9 16 * \log_8 3 &= \log_8\left(\frac{3}{x}\right) & \log_8 9 * \log_8 3 = \log_8\left(\frac{3}{x}\right) \\ \frac{\log_8 16}{\log_8 9} * \log_8 3 &= \log_8\left(\frac{3}{x}\right) & \frac{1 + \frac{1}{3}}{\log_8 9} * \log_8 3 = \log_8\left(\frac{3}{x}\right) \\ \frac{\log_8(8 \cdot 2)}{\log_8 9} * \log_8 3 &= \log_8\left(\frac{3}{x}\right) & \frac{4}{3} \left(\frac{\log_8 3}{\log_8 9}\right) = \log_8\left(\frac{3}{x}\right) \\ \frac{4}{3} \log_8 3 &= \log_8 \frac{3}{x} & \\ \frac{4}{3} \log_9 3 &= \log_8 \frac{3}{x} & \frac{4}{3} \cdot \frac{1}{2} = \log_8\left(\frac{3}{x}\right) \\ && \frac{2}{3} = \log_8\left(\frac{3}{x}\right) \\ && 8^{\frac{2}{3}} = \frac{3}{x} \\ && 4 = \frac{3}{x} \rightarrow \boxed{x = \frac{3}{4}}\end{aligned}$$

answer:  $x = \frac{3}{4}$

\*9) Express  $\log_3 8 + \log_3 6 - \log_3 4 + \log_3 10$  as the log of a single number in simplest form.

$$\log_3\left(\frac{8 \cdot 6 \cdot 10}{4}\right) = \log_3 \frac{480}{4} = \boxed{\log_3 120}$$

answer:  $\log_3 120$

\*10) Evaluate the expression  $\frac{\log_{64} 8 - \log_7 49}{\log_3 \frac{1}{3} + \log_2 (2^{-4})}$

$$= \frac{\frac{1}{2} - 2}{-1 + \log_2\left(\frac{1}{49}\right)} = \frac{\frac{1}{2} - 2}{-1 + -4} = \frac{-\frac{3}{2}}{-5} = \boxed{\frac{3}{10}}$$

answer:  $\frac{3}{10}$

\*\*\*11) If  $\log_2 x + \log_4 x - \log_8 x = 7$ , solve for  $x$ .

$$\begin{aligned}\log_2 x + \frac{\log_2 x}{\log_2 4} - \frac{\log_2 x}{\log_2 8} &= 7 & \log_2(x^{7/6}) = 7 \\ \log_2 x + \frac{\log_2 x}{2} - \frac{\log_2 x}{3} &= 7 & 2^7 = x^{7/6} \\ \log_2 x + \log_2 x^{\frac{1}{2}} - \log_2 x^{\frac{1}{3}} &= 7 & 2^6 = x \\ \log_2\left(\frac{x \cdot x^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right) &= 7 & 64 = x\end{aligned}$$

answer:  $x = 64$