WARM-UP

Factor each completely.

1) $n^2 + 10n + 24$

2) $m^2 - 15m + 56$

3) $2n^2 + 17n + 35$

4) $3x^2 + 29x + 56$

WARM-UP

Factor each completely.

1) $n^2 + 10n + 24$

$(n + 6)(n + 4)$

2) $m^2 - 15m + 56$

$(m - 8)(m - 7)$

3) $2n^2 + 17n + 35$

$(2n + 7)(n + 5)$

4) $3x^2 + 29x + 56$

$(3x + 8)(x + 7)$

COMPLETING THE SQUARE

3.3 Notes

WHY WE COMPLETE THE SQUARE…

The expression $x^2 - 16x + 64$ is called a *perfect square trinomial* because you can factor it to be $(x - 8)(x - 8)$ or $(x - 8)^2$.

If you have an expression missing a *c*-value, such as $x^2 + 14x$, you can add in a *c*-value to make it factorable.

STEPS TO COMPLETE THE SQUARE:

Step 1: Find half of *b*.

Step 2: Square that number.

Step 3: Add the new number on both sides of the equation.

Step 4: Factor the left side and solve.

EXAMPLE 1: Solve $x^2 + 12x - 85 = 0$ by completing the square.

You can't factor $x^2 + 12x - 85 = 0$…yet.

$$x^2 + 12x = 85$$

Step 1: Find half of *b*. $\frac{12}{2} = 6$

Step 2: Square that number. $(6)^2 = 36$

$$x^2 + 12x + 36 = 85 + 36$$

Step 3: Add the new number on both sides of the equation.

EXAMPLE 1: Solve $x^2 + 12x - 85 = 0$ by completing the square.

$$x^2 + 12x + 36 = 85 + 36$$

$$x^2 + 12x + 36 = 121$$

Step 4: Factor the left side and solve.

$$(x + 6)(x + 6) = 121$$

$$(x + 6)^2 = 121$$

$$x + 6 = \pm 11$$

$$x + 6 = 11 \qquad x + 6 = -11$$

$$x = 5 \qquad x = -17$$

EXAMPLE 2: Solve $x^2 - 10x + 7 = 0$ by completing the square.

$x^2 - 10x = -7$

Step 1: Find half of *b*. $\frac{-10}{2} = -5$

Step 2: Square that number. $(-5)^2 = 25$

$x^2 - 10x + 25 = -7 + 25$

Step 3: Add the new number on both sides of the equation.

EXAMPLE 2: Solve $x^2 - 10x + 7 = 0$ by completing the square.

$$x^2 - 10x + 25 = -7 + 25$$

$$x^2 - 10x + 25 = 18$$

Step 4: Factor the left side and solve.

$$(x - 5)(x - 5) = 18$$

$$(x - 5)^2 = 18$$

$$x - 5 = \pm\sqrt{18}$$

$$x - 5 = \pm\sqrt{9 * 2}$$

$$x - 5 = \pm 3\sqrt{2} \longrightarrow x = \pm 3\sqrt{2} + 5$$

EXAMPLE 3: Solve $x^2 - 4x - 8 = 0$ by completing the square.

$x^2 - 4x = 8$

Step 1: Find half of *b*. $\frac{-4}{2} = -2$

Step 2: Square that number. $(-2)^2 = 4$

$x^2 - 4x + 4 = 8 + 4$

Step 3: Add the new number on both sides of the equation.

EXAMPLE 3: Solve $x^2 - 4x - 8 = 0$ by completing the square.

$$x^2 - 4x + 4 = 8 + 4$$

$$x^2 - 4x + 4 = 12$$

Step 4: Factor the left side and solve.

$$(x - 2)(x - 2) = 12$$

$$(x - 2)^2 = 12$$

$$x - 2 = \pm\sqrt{12}$$

$$x - 2 = \pm\sqrt{4 * 3}$$

$$x - 2 = \pm 2\sqrt{3} \longrightarrow x = \pm 2\sqrt{3} + 2$$

EXAMPLE 4: Solve $x^2 + 8x - 5 = 0$ by completing the square.

$x^2 + 8x = 5$

Step 1: Find half of *b*. $\frac{8}{2} = 4$

Step 2: Square that number. $(4)^2 = 16$

$x^2 + 8x + 16 = 5 + 16$

Step 3: Add the new number on both sides of the equation.

EXAMPLE 4: Solve $x^2 + 8x - 5 = 0$ by completing the square.

$$x^2 + 8x + 16 = 5 + 16$$

$$x^2 + 8x + 16 = 21$$

$$(x + 4)(x + 4) = 21$$

$$(x + 4)^2 = 21$$

$$x + 4 = \pm\sqrt{21}$$

$$x = \pm\sqrt{21} - 4$$

Step 4: Factor the left side and solve.

EXAMPLE 5: Solve $x^2 + 6x + 4 = 0$ by completing the square.

$x^2 + 6x = -4$

Step 1: Find half of *b*. $\frac{6}{2} = 3$

Step 2: Square that number. $(3)^2 = 9$

$x^2 + 6x + 9 = -4 + 9$

Step 3: Add the new number on both sides of the equation.

EXAMPLE 5: Solve $x^2 + 6x + 4 = 0$ by completing the square.

$$x^2 + 6x + 9 = -4 + 9$$

$$x^2 + 6x + 9 = 5$$

$$(x + 3)(x + 3) = 5$$

$$(x + 3)^2 = 5$$

$$x + 3 = \pm\sqrt{5}$$

$$x = \pm\sqrt{5} - 3$$

Step 4: Factor the left side and solve.