

Solving with Factoring & Identities - Round 2

Factor to solve each equation for  $0 \leq \theta < 2\pi$ .

$$1) 3\tan \theta + 2\sqrt{3}\tan \theta \cos \theta - \cos \theta = -\cos \theta$$

$$3\tan \theta + 2\sqrt{3}\tan \theta \cos \theta = 0$$

$$\tan \theta (3 + 2\sqrt{3} \cos \theta) = 0$$

$$\tan \theta = 0$$

$$\theta = \tan^{-1}(0)$$

#1:  $\theta = 0$

#2:  $0 + \pi = \pi$

$$3 + 2\sqrt{3} \cos \theta = 0$$

$$2\sqrt{3} \cos \theta = -3$$

$$\cos \theta = \frac{-3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

#1:  $\theta = \frac{5\pi}{6}$

#2:  $2\pi - \frac{5\pi}{6} = \frac{7\pi}{6}$

$$2) 2\tan \theta \cos \theta - 2\cos \theta = \sqrt{3}\tan \theta - 2\cos \theta$$

$$2\tan \theta \cos \theta = \sqrt{3}\tan \theta$$

$$2\tan \theta \cos \theta - \sqrt{3}\tan \theta = 0$$

$$\tan \theta (2\cos \theta - \sqrt{3}) = 0$$

$$\tan \theta = 0$$

$$\theta = \tan^{-1}(0)$$

#1:  $\theta = 0$

#2:  $0 + \pi = \pi$

$$2\cos \theta - \sqrt{3} = 0$$

$$2\cos \theta = \sqrt{3}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

#1:  $\theta = \pi/6$

#2:  $2\pi - \pi/6 = \frac{11\pi}{6}$

$$3) \sqrt{3}\csc^2 \theta = -2\csc \theta$$

$$\sqrt{3}\csc^2 \theta + 2\csc \theta = 0$$

$$\csc \theta (\sqrt{3}\csc \theta + 2) = 0$$

$$\csc \theta = 0$$

$$\frac{1}{\sin \theta} = 0$$

$$\sin \theta = \emptyset$$

$$\theta = \sin^{-1}(\emptyset)$$

NO SOLUTION

$$\sqrt{3}\csc \theta + 2 = 0$$

$$\sqrt{3}\csc \theta = -2$$

$$\csc \theta = -\frac{2}{\sqrt{3}}$$

$$\frac{1}{\sin \theta} = -\frac{2}{\sqrt{3}}$$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

#1:  $\theta = \frac{-\pi}{3} \rightarrow \frac{5\pi}{3}$

#2:  $\pi - \frac{-\pi}{3} = \frac{4\pi}{3}$

$$4) -\sec^2 \theta = \sqrt{2}\sec \theta - 2\sec^2 \theta$$

$$-\sec^2 \theta + 2\sec^2 \theta = \sqrt{2}\sec \theta$$

$$\sec^2 \theta - \sqrt{2}\sec \theta = 0$$

$$\sec \theta (\sec \theta - \sqrt{2}) = 0$$

$$\sec \theta = 0$$

$$\frac{1}{\cos \theta} = 0$$

$$\cos \theta = \emptyset$$

$$\theta = \cos^{-1}(\emptyset)$$

NO SOLUTION

$$\sec \theta - \sqrt{2} = 0$$

$$\sec \theta = \sqrt{2}$$

$$\frac{1}{\cos \theta} = \sqrt{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

#1:  $\theta = \frac{\pi}{4}$

#2:  $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Use a Pythagorean Identity to solve each equation for  $0 \leq \theta < 2\pi$ .

5)  $2\cot \theta = \csc^2 \theta$

$$2\cot \theta - \csc^2 \theta = 0$$

$$2\cot \theta - (1 + \cot^2 \theta) = 0$$

$$2\cot \theta - 1 - \cot^2 \theta = 0$$

$$-\cot^2 \theta + 2\cot \theta - 1 = 0$$

$$(-\cot \theta + 1)(\cot \theta - 1) = 0$$

$$\downarrow \qquad \qquad \downarrow$$

$$-\cot \theta + 1 = 0 \quad \cot \theta - 1 = 0$$

$$-\cot \theta = -1 \quad \cot \theta = 1$$

$$\cot \theta = 1 \quad \leftarrow \text{SAME}$$

$$\frac{1}{\tan \theta} = 1$$

$$\tan \theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = \pi/4$$

$$\frac{\pi}{4} + \pi = \frac{5\pi}{4}$$

6)  $\csc \theta + 1 = \cot^2 \theta$

$$\csc \theta + 1 = \csc^2 \theta - 1$$

$$\csc^2 \theta - \csc \theta - 2 = 0$$

$$(\csc \theta - 2)(\csc \theta + 1) = 0$$

$$\downarrow \qquad \qquad \downarrow$$

$$\csc \theta - 2 = 0 \qquad \csc \theta + 1 = 0$$

$$\csc \theta = 2 \qquad \csc \theta = -1$$

$$\frac{1}{\sin \theta} = 2 \qquad \frac{1}{\sin \theta} = -1$$

$$\sin \theta = \frac{1}{2} \qquad \sin \theta = -1$$

$$\theta = \sin^{-1}(\frac{1}{2})$$

$$\theta = \sin^{-1}(-1)$$

#1:  $\theta = \pi/6$   $\xrightarrow{+\pi}$   $\frac{3\pi}{2}$

#2:  $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

#2:  $\pi - \frac{\pi}{2} = \frac{3\pi}{2}$   $\leftarrow$  SAME

7)  $-3\csc \theta = -\cot^2 \theta - 3$

$$\cot^2 \theta - 3\csc \theta + 3 = 0$$

$$\csc^2 \theta - 1 - 3\csc \theta + 3 = 0$$

$$\csc^2 \theta - 3\csc \theta + 2 = 0$$

$$(\csc \theta - 2)(\csc \theta - 1) = 0$$

$$\downarrow \qquad \qquad \downarrow$$

$$\csc \theta - 2 = 0 \qquad \csc \theta - 1 = 0$$

$$\csc \theta = 2 \qquad \csc \theta = 1$$

$$\frac{1}{\sin \theta} = 2 \qquad \frac{1}{\sin \theta} = 1$$

$$\sin \theta = \frac{1}{2} \qquad \sin \theta = 1$$

$$\theta = \sin^{-1}(\frac{1}{2}) \qquad \theta = \sin^{-1}(1)$$

$\theta = \pi/6$   $\leftarrow$  SAME

$\pi - \pi/6 = \frac{5\pi}{6}$   $\pi - \pi/2 = \frac{\pi}{2}$

8)  $-1 + \csc \theta = -\cot^2 \theta$

$$\cot^2 \theta + \csc \theta - 1 = 0$$

$$\csc^2 \theta - 1 + \csc \theta - 1 = 0$$

$$\csc^2 \theta + \csc \theta - 2 = 0$$

$$(\csc \theta + 2)(\csc \theta - 1) = 0$$

$$\downarrow \qquad \qquad \downarrow$$

$$\csc \theta + 2 = 0 \qquad \csc \theta - 1 = 0$$

$$\csc \theta = -2 \qquad \csc \theta = 1$$

$$\frac{1}{\sin \theta} = -2 \qquad \frac{1}{\sin \theta} = 1$$

$$\sin \theta = -\frac{1}{2} \qquad \sin \theta = 1$$

$$\theta = \sin^{-1}(-\frac{1}{2})$$

$$\theta = \sin^{-1}(1)$$

$\theta = -\pi/6$   $\xrightarrow{+\pi}$   $\frac{11\pi}{6}$

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\theta = \pi/2$   $\leftarrow$  SAME

$\pi - \pi/2 = \pi/2$