Honors Precalculus - Unit 3

Name KEY

Solving with Factoring & Identities

Date________________ Period____

Factor to solve each equation for $0 \le \theta < 2\pi$.

1) $-2\sqrt{3}\tan\theta\sin\theta + 3\tan\theta + 2\sin\theta = 2\sin\theta$

$-2\sin\theta \quad -2\sin\theta$

$-2\sqrt{3}\tan\theta\sin\theta + 3\tan\theta = 0$

$\tan\theta\left(-2\sqrt{3}\sin\theta + 3\right) = 0$

$\tan\theta = 0$

$\theta = \tan^{-1}(0)$

$\boxed{\theta = 0}$

$0 + \pi = \boxed{\pi}$

$+\pi = \cancel{2\pi}$

$-2\sqrt{3}\sin\theta + 3 = 0$

$-2\sqrt{3}\sin\theta = -3$

$\sin\theta = \frac{-3}{-2\sqrt{3}}$

$\sin\theta = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\theta = \frac{\pi}{3}}$

$\pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

2) $0 = -3\tan\theta + \sqrt{3}\tan^2\theta$

$0 = \tan\theta(-3 + \sqrt{3}\tan\theta)$

$0 = \tan\theta$

$\tan^{-1}(0) = \theta$

$\boxed{0 = \theta}$

$0 + \pi = \boxed{\pi}$

$-3 + \sqrt{3}\tan\theta = 0$

$\sqrt{3}\tan\theta = 3$

$\tan\theta = \frac{3}{\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\boxed{\theta = \frac{\pi}{3}}$

$\frac{\pi}{3} + \pi = \boxed{\frac{4\pi}{3}}$

3) $\cos\theta - 2\cos^2\theta = \sqrt{2}\cos^2\theta - 2\cos^2\theta$

$-\cos\theta + 2\cos^2\theta \quad -\cos\theta + 2\cos^2\theta$

$0 = \sqrt{2}\cos^2\theta - \cos\theta$

$0 = \cos\theta(\sqrt{2}\cos\theta - 1)$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\boxed{\theta = \frac{\pi}{2}}$

$2\pi - \frac{\pi}{2} = \boxed{\frac{3\pi}{2}}$

$\sqrt{2}\cos\theta - 1 = 0$

$\sqrt{2}\cos\theta = 1$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\theta = \frac{\pi}{4}}$

$2\pi - \frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$

4) $2\cos\theta\sin\theta - \sin\theta = \sqrt{3}\cos\theta - \sin\theta$

$-\sqrt{3}\cos\theta + \sin\theta \quad -\sqrt{3}\cos\theta + \sin\theta$

$2\cos\theta\sin\theta - \sqrt{3}\cos\theta = 0$

$\cos\theta(2\sin\theta - \sqrt{3}) = 0$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\boxed{\theta = \pi/2}$

$2\pi - \frac{\pi}{2} = \boxed{\frac{3\pi}{2}}$

$2\sin\theta - \sqrt{3} = 0$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\theta = \frac{\pi}{3}}$

$\pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

Use a Pythagorean Identity to solve each equation for $0 \le \theta < 2\pi$.

5) $\cos^2\theta + \cos\theta = \sin^2\theta$

$\cos^2\theta + \cos\theta = 1 - \cos^2\theta$

$+\cos^2\theta \quad -1 \qquad\qquad -1 \quad +\cos^2\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0$

$\cos\theta = 1/2$

$\theta = \cos^{-1}(1/2)$

$\boxed{\theta = \pi/3}$

$2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$

$\cos\theta + 1 = 0$

$\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$\boxed{\theta = \pi}$

$2\pi - \pi = \pi$

6) $-\sin^2\theta + 3\sin\theta = -\cos^2\theta + 2$

$-\sin^2\theta + 3\sin\theta = \sin^2\theta - 1 + 2$

$-\sin^2\theta + 3\sin\theta = \sin^2\theta + 1$

$+\sin^2\theta \quad -3\sin\theta \qquad +\sin^2\theta - 3\sin\theta$

$0 = 2\sin^2\theta - 3\sin\theta + 1$

$0 = (2\sin\theta - 1)(\sin\theta - 1)$

$2\sin\theta - 1 = 0$

$\sin\theta = 1/2$

$\theta = \sin^{-1}(1/2)$

$\theta = \boxed{\frac{\pi}{6}}$

$\pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

$\sin\theta + 1 = 0$

$\sin\theta = -1$

$\theta = \sin^{-1}(-1)$

$\theta = \frac{-\pi}{2} \xrightarrow{+2\pi} \boxed{\frac{\pi}{2}}$

7) $-\sin\theta = \cos^2\theta - \sin^2\theta$

$-\sin\theta = 1 - \sin^2\theta - \sin^2\theta$

$0 = 1 - 2\sin^2\theta - \sin\theta$

$0 = -2\sin^2\theta - \sin\theta + 1$

$0 = (-2\sin\theta - 1)(\sin\theta - 1)$

$-2\sin\theta - 1 = 0$

$\sin\theta = -1/2$

$\theta = \sin^{-1}(-\frac{1}{2})$

$\theta = -\frac{\pi}{6} \xrightarrow{+2\pi} \boxed{\frac{11\pi}{6}}$

$\pi - \frac{-\pi}{6} = \boxed{\frac{7\pi}{6}}$

$\sin\theta - 1 = 0$

$\sin\theta = 1$

$\theta = \sin^{-1}(1)$

$\boxed{\theta = \pi/2}$

$\pi - \pi/2 = \pi/2$

8) $2 - \cos^2\theta + 2\sin\theta = 0$

$2 - (1 - \sin^2\theta) + 2\sin\theta = 0$

$2 - 1 + \sin^2\theta + 2\sin\theta = 0$

$1 + \sin^2\theta + 2\sin\theta = 0$

$\sin^2\theta + 2\sin\theta + 1 = 0$

$(\sin\theta + 1)^2 = 0$

$\sin\theta + 1 = \sqrt{0}$

$\sin\theta + 1 = 0$

$\sin\theta = -1$

$\theta = \sin^{-1}(-1)$

$\theta = -\frac{\pi}{2} \xrightarrow{+2\pi} \boxed{\frac{3\pi}{2}}$