

Solving with Factoring & Identities - NOTES

Factor to solve each equation for $0 \leq \theta < 2\pi$.

$$1) \begin{aligned} 3\sin\theta - 3\sin^2\theta &= -2\sqrt{3}\sin^2\theta - 3\sin^2\theta \\ &\quad + 3\sin^2\theta \quad + 3\sin^2\theta \\ 3\sin\theta &= -2\sqrt{3}\sin^2\theta \\ 2\sqrt{3}\sin^2\theta + 3\sin\theta &= 0 \\ \sin\theta(2\sqrt{3}\sin\theta + 3) &= 0 \end{aligned}$$

(GCF)

$$\begin{aligned} \sin\theta = 0 & \quad 2\sqrt{3}\sin\theta + 3 = 0 \\ \theta = \sin^{-1}(0) & \quad 2\sqrt{3}\sin\theta = -3 \\ \theta = 0 & \quad 2\sqrt{3}\sin\theta = -3 \\ & \quad \sin\theta = \frac{-3}{2\sqrt{3}} \\ & \quad \sin\theta = \frac{-3\sqrt{3}}{6} = \frac{-\sqrt{3}}{2} \\ & \quad \theta = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) \\ & \quad \theta = -\frac{\pi}{3} \xrightarrow{+2\pi} \frac{5\pi}{3} \\ & \quad \pi - \frac{-\pi}{3} = \frac{4\pi}{3} \end{aligned}$$

$$\theta = 0$$

$$\pi - 0 = \pi$$

$$2) \begin{aligned} 2\sin\theta + \sin\theta \tan\theta &= \sin\theta \\ -\sin\theta & \quad -\sin\theta \\ \sin\theta + \sin\theta \tan\theta &= 0 \\ \sin\theta(1 + \tan\theta) &= 0 \\ \downarrow & \quad \downarrow \\ \sin\theta = 0 & \quad 1 + \tan\theta = 0 \\ \theta = \sin^{-1}(0) & \quad \tan\theta = -1 \\ \theta = 0 & \quad \theta = \tan^{-1}(-1) \\ \pi - 0 = \pi & \quad \theta = \frac{-\pi}{4} \xrightarrow{+\pi} \frac{3\pi}{4} \\ & \quad \frac{3\pi}{4} + \pi = \frac{7\pi}{4} \end{aligned}$$

$$3) \begin{aligned} \sqrt{3}\sin\theta + 2\sin\theta \cos\theta + 3\sin\theta &= 3\sin\theta \\ & \quad -3\sin\theta \quad -3\sin\theta \\ \sqrt{3}\sin\theta + 2\sin\theta \cos\theta &= 0 \\ \sin\theta(\sqrt{3} + 2\cos\theta) &= 0 \\ \downarrow & \quad \downarrow \\ \sin\theta = 0 & \quad \sqrt{3} + 2\cos\theta = 0 \\ \theta = 0 & \quad 2\cos\theta = -\sqrt{3} \\ \pi - 0 = \pi & \quad \cos\theta = \frac{-\sqrt{3}}{2} \\ & \quad \theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) \\ & \quad \theta = \frac{5\pi}{6} \\ & \quad 2\pi - \frac{5\pi}{6} = \frac{7\pi}{6} \end{aligned}$$

$$4) \begin{aligned} \tan\theta \cos\theta + \tan\theta + 3\cos\theta &= 3\cos\theta \\ & \quad -3\cos\theta \quad -3\cos\theta \\ \tan\theta \cos\theta + \tan\theta &= 0 \\ \tan\theta(\cos\theta + 1) &= 0 \\ \downarrow & \quad \downarrow \\ \tan\theta = 0 & \quad \cos\theta + 1 = 0 \\ \theta = \tan^{-1}(0) & \quad \cos\theta = -1 \\ \theta = 0 & \quad \theta = \cos^{-1}(-1) \\ +\pi & \quad \theta = \pi \\ \theta = \pi & \quad 2\pi - \pi = \pi \end{aligned}$$

Use a Pythagorean Identity to solve each equation for $0 \leq \theta < 2\pi$.

5) $2 + \cos^2 \theta = -3\cos \theta + \sin^2 \theta$

$$2 + \cos^2 \theta = -3\cos \theta + 1 - \cos^2 \theta$$

$$+ 3\cos \theta - 1 + \cos^2 \theta$$

$$2\cos^2 \theta + 3\cos \theta + 1 = 0$$

$$(2\cos \theta + 1)(\cos \theta + 1) = 0$$

$$\downarrow$$

$$2\cos \theta + 1 = 0$$

$$2\cos \theta = -1$$

$$\cos \theta = -1/2$$

$$\theta = \cos^{-1}(-1/2)$$

$$\boxed{\theta = \frac{2\pi}{3}}$$

$$2\pi - \frac{2\pi}{3} = \boxed{\frac{4\pi}{3}}$$

$$\downarrow$$

$$\cos \theta + 1 = 0$$

$$\cos \theta = -1$$

$$\theta = \cos^{-1}(-1)$$

$$\boxed{\theta = \pi}$$

$$2\pi - \pi = \pi$$

6) $\cos^2 \theta - 2 = 3\sin^2 \theta + 4\sin \theta$

$$1 - \sin^2 \theta - 2 = 3\sin^2 \theta + 4\sin \theta$$

$$-\sin^2 \theta = 3\sin^2 \theta + 4\sin \theta$$

$$+ \sin^2 \theta + \sin^2 \theta$$

$$0 = 4\sin^2 \theta + 4\sin \theta + 1$$

$$0 = (2\sin \theta + 1)(2\sin \theta + 1)$$

$$0 = (2\sin \theta + 1)^2$$

$$0 = 2\sin \theta + 1$$

$$-1 = 2\sin \theta$$

$$-1/2 = \sin \theta$$

$$\sin^{-1}(-1/2) = \theta$$

$$-\frac{\pi}{6} = \theta \xrightarrow{+2\pi} \boxed{\frac{11\pi}{6}}$$

$$\pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$$

7) $\sin^2 \theta + \cos \theta = \cos^2 \theta$

$$1 - \cos^2 \theta + \cos \theta = \cos^2 \theta$$

$$-2\cos^2 \theta + \cos \theta + 1 = 0$$

$$(2\cos \theta + 1)(-\cos \theta + 1) = 0$$

$$\downarrow$$

$$2\cos \theta + 1 = 0$$

$$2\cos \theta = -1$$

$$\cos \theta = -1/2$$

$$\boxed{\theta = \frac{2\pi}{3}}$$

$$2\pi - \frac{2\pi}{3} = \boxed{\frac{4\pi}{3}}$$

$$\downarrow$$

$$-\cos \theta + 1 = 0$$

$$-\cos \theta = -1$$

$$\cos \theta = 1$$

$$\theta = \cos^{-1}(1)$$

$$\boxed{\theta = 0}$$

8) $-2\cot \theta = -\csc^2 \theta$

$$-2\cot \theta = -(1 + \cot^2 \theta)$$

$$-2\cot \theta = -1 - \cot^2 \theta$$

$$\cot^2 \theta - 2\cot \theta + 1 = 0$$

$$(\cot \theta - 1)(\cot \theta - 1) = 0$$

$$(\cot \theta - 1)^2 = 0$$

$$\cot \theta - 1 = 0$$

$$\cot \theta = 1$$

$$\frac{1}{\tan \theta} = 1$$

$$\tan \theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\boxed{\theta = \frac{\pi}{4}}$$

$$+ \pi$$

$$= \boxed{\frac{5\pi}{4}}$$