

Warm-Up

Simplify each expression.

$$1) \frac{18}{12x + 12}$$

$$2) \frac{6x^2 + 42x}{x + 7}$$

$$3) \frac{a^2 + 2a - 8}{a + 4}$$

$$4) \frac{x + 4}{5x + 20}$$

Warm-Up

Simplify each expression.

$$1) \frac{18}{12x + 12}$$
$$\frac{3}{2(x + 1)}$$

$$3) \frac{a^2 + 2a - 8}{a + 4}$$
$$a - 2$$

$$2) \frac{6x^2 + 42x}{x + 7}$$
$$6x$$

$$4) \frac{x + 4}{5x + 20}$$
$$\frac{1}{5}$$

5-6 Notes

Laws of Logarithms

Learning Targets:

- I can use properties of logs to evaluate or rewrite log expressions.
- I can use use properties of logs to expand or condense expressions.
- I can use log functions to model and solve real-life applications.

Properties of Logs

Condensed Form

Expanded Form

$$\log_b(xy) \longleftrightarrow \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) \longleftrightarrow \log_b x - \log_b y$$

$$\log_b x^y \longleftrightarrow y \times \log_b x$$

Properties of Logs

Also...

$$\log_b x = \log_b y \quad \text{if and only if} \quad x = y$$

Examples: Expand each logarithm

$$1) \log_6 \left(\frac{5}{d} \right) = \log_6 5 - \log_6 d$$

$$2) \log_3 7^k = k \log_3 7$$

$$3) \ln (a \times 54) = \ln a + \ln 54$$

Examples: Condense each logarithm

$$1) 16 \log_4 w = \log_4 w^{16}$$

$$2) \ln a + \ln 4 = \ln(4a)$$

$$3) \log_8 b - \log_8 17 = \log_8 \left(\frac{b}{17} \right)$$

Combining Properties:

1) Condense: $3\log_{1.5} 5 + 6\log_{1.5} 4$

$$= \log_{1.5} 5^3 + \log_{1.5} 4^6 = \log_{1.5} (5^3 \times 4^6)$$

2) Expand: $\log_4 \left(\frac{7}{19}\right)^3 = 3\log_4 \left(\frac{7}{19}\right)$

$$= 3\log_4 7 - 3\log_4 19$$

Fractions and Radicals...

$$\frac{x}{5} = \frac{1}{5}x$$

$$\frac{x}{7} = \frac{1}{7}x$$

$$x^{\frac{1}{4}} = \sqrt[4]{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$\frac{\log_6 x}{9} = \frac{1}{9} \log_6 x \longrightarrow \log_6 x^{\frac{1}{9}} = \log_6 \sqrt[9]{x}$$

You try...

Condense each expression.

1) $16\log_6 x - 4\log_6 y$

$$\log_6 \left(\frac{x^{16}}{y^4} \right)$$

2) $2\ln z + \frac{\ln x}{3}$

$$\ln z^2 \sqrt[3]{x}$$

3) $3\log_4 a + 18\log_4 b$ $\log_4 a^3 b^{18}$

Expand each logarithm.

4) $\log_9 \left(\frac{x^2}{y} \right)^4$ $8\log_9 x - 4\log_9 y$

5) $\log_4 (x^3 \cdot y)^5$ $15\log_4 x + 5\log_4 y$

6) $\log_5 (xy^6)^5$ $5\log_5 x + 30\log_5 y$

More Examples:

1) Express $\log_b MN^2$ in terms of $\log_b M$ and $\log_b N$.

$$\begin{aligned}\log_b MN^2 &= \log_b M + \log_b N^2 \\ &= \log_b M + 2\log_b N\end{aligned}$$

More Examples:

2) Express $\log_b \sqrt{\frac{M^3}{N}}$ in terms of $\log_b M$ and $\log_b N$.

$$\begin{aligned}\log_b \sqrt{\frac{M^3}{N}} &= \log_b \left(\frac{M^3}{N} \right)^{\frac{1}{2}} \\ &= \frac{1}{2} \log_b \left(\frac{M^3}{N} \right) \\ &= \frac{1}{2} \log_b M^3 - \frac{1}{2} \log_b N \\ &= \frac{3}{2} \log_b M - \frac{1}{2} \log_b N\end{aligned}$$

More Examples:

3) Simplify $\log 45 - 2 \log 3$.

$$\log 45 - 2 \log 3 = \log 45 - \log 3^2$$

$$= \log 45 - \log 9$$

$$= \log \frac{45}{9}$$

$$= \log 5$$

More Examples:

4) Express y in terms of x if $\ln y = \frac{1}{3}\ln x + \ln 4$.

$$\ln y = \ln x^{\frac{1}{3}} + \ln 4$$

$$\ln y = \ln 4x^{\frac{1}{3}}$$

$$y = 4x^{\frac{1}{3}}$$

$$y = 4\sqrt[3]{x}$$

More Examples:

5) Solve $\log_2 x + \log_2(x - 2) = 3$.

$$\log_2 x + \log_2(x - 2) = 3$$

$$\log_2 x(x - 2) = 3$$

Rewrite in exponential form

$$2^3 = x(x - 2)$$

$$2^3 = x(x - 2)$$

$$8 = x^2 - 2x$$

$$0 = x^2 - 2x - 8$$

$$0 = (x - 4)(x + 2)$$

$$0 = x - 4$$

$$0 = x + 2$$

$$4 = x$$

$$-2 = x$$

Practice Problems

Page 200

#1-34