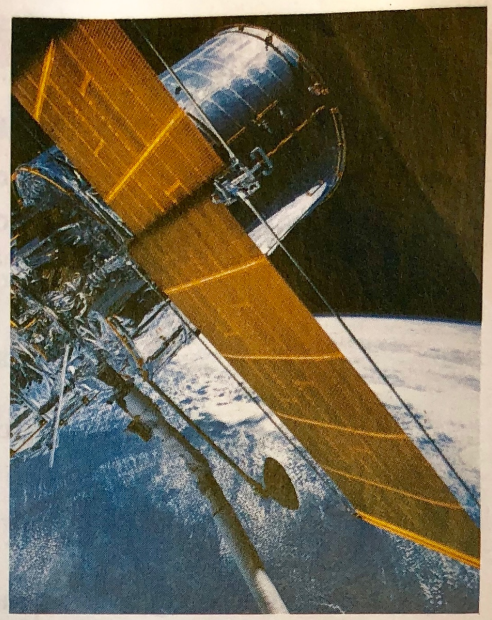
From Example 4 and the definitions of $\sin\theta$ and $\cos\theta$, you can see that the sine and cosine functions repeat their values every 360° or 2π radians. Formally this means that for all θ:

$$\sin(\theta + 360°) = \sin\theta$$
$$\cos(\theta + 360°) = \cos\theta$$

$$\sin(\theta + 2\pi) = \sin\theta$$
$$\cos(\theta + 2\pi) = \cos\theta$$

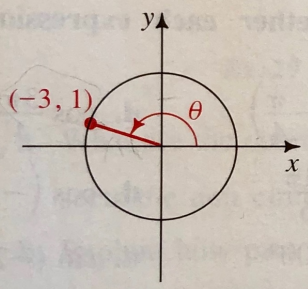
We summarize these facts by saying that the sine and cosine functions are *periodic* and that they have a *fundamental period* of 360°, or 2π radians. It is the periodic nature of these functions that makes them useful in describing many repetitive phenomena such as tides, sound waves, and the orbital paths of satellites.



CLASS EXERCISES

Find $\sin\theta$ and $\cos\theta$.

1.



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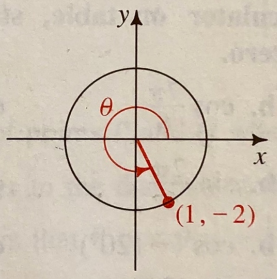

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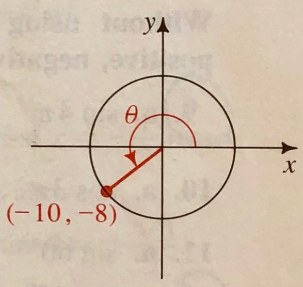


4. State whether each expression is positive or negative.

a. $\sin 165°$ **b.** $\sin 265°$ **c.** $\cos 210°$ **d.** $\cos 310°$

e. $\sin\frac{5\pi}{6}$ **f.** $\cos\frac{5\pi}{6}$ **g.** $\sin\frac{4\pi}{3}$ **h.** $\cos\frac{5\pi}{3}$

i. $\sin 2$ **j.** $\cos 2$ **k.** $\sin 4$ **l.** $\cos 4$

5. Does $\cos\theta$ increase or decrease as:

a. θ increases from 0° to 90°? decrease **b.** θ increases from 90° to 180°?

c. θ increases from 180° to 270°? **d.** θ increases from 270° to 360°?

6. Answer Exercise 5 for $\sin\theta$.

7. Use the unit circle to justify the fact that for all θ:

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

8. There are infinitely many values of θ for which $\cos\theta = 0$. Name several.

9. a. Explain the meaning of $\theta = 45° + n \cdot 360°$, where n is an integer.

b. What is the equivalent statement if θ is expressed in radians?

WRITTEN EXERCISES

Find the value of each expression without using a calculator or table.

A

1. **a.** $\sin 180°$ **b.** $\cos 180°$ **c.** $\sin 270°$ **d.** $\cos 270°$
2. **a.** $\sin(-90°)$ **b.** $\cos(-90°)$ **c.** $\sin 360°$ **d.** $\cos 360°$
3. **a.** $\sin(-\pi)$ **b.** $\cos \pi$ **c.** $\sin \frac{3\pi}{2}$ **d.** $\cos \frac{\pi}{2}$
4. **a.** $\cos 2\pi$ **b.** $\sin\left(-\frac{\pi}{2}\right)$ **c.** $\sin 3\pi$ **d.** $\cos\left(-\frac{3\pi}{2}\right)$

Name each quadrant described.

5. **a.** $\sin\theta > 0$ and $\cos\theta < 0$ **b.** $\sin\theta < 0$ and $\cos\theta < 0$
6. **a.** $\sin\theta < 0$ and $\cos\theta > 0$ **b.** $\sin\theta > 0$ and $\sin(90° + \theta) > 0$

Without using a calculator or table, solve each equation for *all* θ in radians.

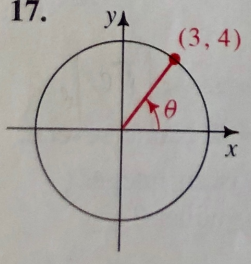
7. **a.** $\sin\theta = 1$ **b.** $\cos\theta = -1$ **c.** $\sin\theta = 0$ **d.** $\sin\theta = 2$
8. **a.** $\cos\theta = 1$ **b.** $\sin\theta = -1$ **c.** $\cos\theta = 0$ **d.** $\cos\theta = -3$

Without using a calculator or table, state whether each expression is positive, negative, or zero.

9. **a.** $\sin 4\pi$ **b.** $\cos \frac{7\pi}{6}$ **c.** $\sin\left(-\frac{\pi}{4}\right)$ **d.** $\cos \frac{3\pi}{4}$
10. **a.** $\cos 3\pi$ **b.** $\sin \frac{2\pi}{3}$ **c.** $\sin \frac{11\pi}{6}$ **d.** $\cos\left(-\frac{\pi}{2}\right)$
11. **a.** $\sin 60°$ **b.** $\cos(-120°)$ **c.** $\cos 300°$ **d.** $\sin(-210°)$
12. **a.** $\cos 45°$ **b.** $\sin 135°$ **c.** $\cos(-225°)$ **d.** $\sin(-315°)$
13. **a.** $\sin \frac{7\pi}{4}$ **b.** $\sin\left(-\frac{\pi}{6}\right)$ **c.** $\cos \frac{3\pi}{2}$ **d.** $\cos \frac{\pi}{3}$
14. **a.** $\cos\left(-\frac{\pi}{3}\right)$ **b.** $\sin \frac{\pi}{6}$ **c.** $\sin \frac{5\pi}{4}$ **d.** $\cos \frac{7\pi}{4}$
15. **a.** $\cos 89°$ **b.** $\cos 91°$ **c.** $\sin 720°$ **d.** $\sin(-270°)$
16. **a.** $\sin 1°$ **b.** $\sin(-1°)$ **c.** $\cos 90°$ **d.** $\cos 540°$

Find $\sin\theta$ and $\cos\theta$.

17.



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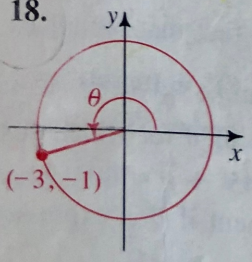

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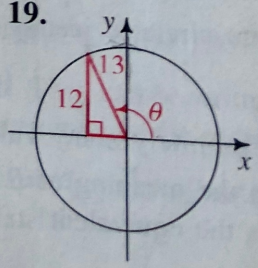

20.