Honors Precalculus - Midterm Exam Name_____

Unit 3 Review

Date_____ Period____

Use identities to find the value of each expression.

1) Find $\sin\theta$ and $\tan\theta$ if $\sec\theta = 2$ and $\csc\theta < 0$.

$-\frac{\sqrt{3}}{2}$ and $-\sqrt{3}$

2) Find $\csc\theta$ and $\tan\theta$ if $\sec\theta = 4$ and $\sin\theta < 0$.

$-\frac{4\sqrt{15}}{15}$ and $-\sqrt{15}$

Verify each identity.

3) $\frac{\sin^2 x}{\cot^2 x} = \frac{\tan^2 x}{\csc^2 x}$

$\frac{\sin^2 x}{\cot^2 x}$ Use $\cot x = \frac{1}{\tan x}$

$\sin^2 x\tan^2 x$ Use $\csc x = \frac{1}{\sin x}$

$\frac{\tan^2 x}{\csc^2 x}$ ■

4) $\sin x\cot x = \cos x$

$\sin x\cot x$ Use $\cot x = \frac{\cos x}{\sin x}$

$\frac{\sin x\cos x}{\sin x}$ Cancel common factors

$\cos x$ ■

5) $\cot^2 x - \tan^2 x = \csc^2 x - \sec^2 x$

$\cot^2 x - \tan^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

$\csc^2 x - \tan^2 x - 1$ Use $\tan^2 x + 1 = \sec^2 x$

$\csc^2 x - \sec^2 x$ ■

6) $\csc x\sin x\tan^2 x = \sec^2 x - 1$

$\csc x\sin x\tan^2 x$ Use $\csc x = \frac{1}{\sin x}$

$\frac{\sin x\tan^2 x}{\sin x}$ Cancel common factors

$\tan^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$\sec^2 x - 1$ ■

7) $\dfrac{\tan x \sec x}{\sin x} = \tan^2 x + 1$

$\dfrac{\tan x \sec x}{\sin x}$

$\dfrac{\sec x \sin x}{\sin x \cos x}$

$\dfrac{\sec x}{\cos x}$

$\sec^2 x$

$\tan^2 x + 1$ ■

8) $\cos x \cot^2 x \sec x = \csc^2 x - 1$

Use $\tan x = \dfrac{\sin x}{\cos x}$

$\cos x \cot^2 x \sec x$ Use $\sec x = \dfrac{1}{\cos x}$

Cancel common factors

$\dfrac{\cos x \cot^2 x}{\cos x}$ Cancel common factors

Use $\sec x = \dfrac{1}{\cos x}$

$\cot^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

Use $\tan^2 x + 1 = \sec^2 x$

$\csc^2 x - 1$ ■

9) $\dfrac{\tan^2 x}{\sin^2 x} = \tan^2 x + 1$

$\dfrac{\tan^2 x}{\sin^2 x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\dfrac{\sin^2 x}{\sin^2 x \cos^2 x}$ Cancel common factors

$\dfrac{1}{\cos^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$\sec^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$\tan^2 x + 1$ ■

Write each product as a sum or difference.

10) $-3\sin 77° \sin 40°$

$\dfrac{-3\cos 37° + 3\cos 117°}{2}$

11) $\cos 70° \sin 63°$

$\dfrac{\sin 133° - \sin 7°}{2}$

12) $4\sin 5A\cos 4A$

$2\sin 9A + 2\sin A$

Write each sum or difference as a product.

13) $\cos 35° + \cos 241°$

$2\cos 138°\cos 103°$

14) $-2(\sin 158° + \sin 118°)$

$-4\sin 138°\cos 20°$

15) $\cos 7\theta + \cos 13\theta$

$2\cos 10\theta\cos 3\theta$

Use the sum and difference formulas to find the exact value of each.

16) $\sin 105°$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

17) $\sin \frac{5\pi}{12}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

18) $\cos 195°$

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

19) $\cos \frac{19\pi}{12}$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

20) $\tan 165°$

$\sqrt{3}-2$

21) $\tan \frac{11\pi}{12}$

$\sqrt{3}-2$

Use the half-angle identities to find the exact value of each.

22) $\sin 285°$

$$-\frac{\sqrt{2+\sqrt{3}}}{2}$$

23) $\cos 165°$

$$-\frac{\sqrt{2+\sqrt{3}}}{2}$$

24) $\tan 202.5°$

$$\sqrt{2}-1$$

Verify each identity.

25) $\dfrac{2\sin^2 x}{1+\cos 2x} = \tan^2 x$

$\dfrac{2\sin^2 x}{1+\cos 2x}$ Use $\cos^2 x = \dfrac{1+\cos 2x}{2}$

$\dfrac{\sin^2 x}{\cos^2 x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\tan^2 x$ ■

26) $\dfrac{\sin 2x}{\sec^2 x} = 2\sin x\cos^3 x$

$\dfrac{\sin 2x}{\sec^2 x}$ Use $\sin 2x = 2\sin x\cos x$

$\dfrac{2\sin x\cos x}{\sec^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$2\sin x\cos^3 x$ ■

27) $\dfrac{\cos 2x}{2\sin x\cos x} = \dfrac{1}{\tan 2x}$

$\dfrac{\cos 2x}{2\sin x\cos x}$ Use $\sin 2x = 2\sin x\cos x$

$\dfrac{\cos 2x}{\sin 2x}$ Use $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$

$\dfrac{1}{\tan 2x}$ ■

28) $\dfrac{2\cos^2 x}{1 - \cos 2x} = \cot^2 x$

$\dfrac{2\cos^2 x}{1 - \cos 2x}$ Use $\sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\dfrac{\cos^2 x}{\sin^2 x}$ Use $\cot x = \dfrac{\cos x}{\sin x}$

$\cot^2 x$ ■

Solve each equation for $0 \le \theta < 2\pi$.

29) $-\sqrt{3}\sin\theta - \sin\theta = -3\sin\theta\tan\theta - \sin\theta$

$\left\{0, \frac{\pi}{6}, \pi, \frac{7\pi}{6}\right\}$

30) $-4\sin^2\theta = \sin\theta - 2\sin^2\theta$

$\left\{0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

31) $5 = -\tan^2\theta + 4\sec\theta$

$\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

32) $-\cos^2\theta + 4\sin\theta = -3\sin^2\theta - 2$

$\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

33) $-\sin^2 2\theta = -2\sin^2 2\theta + 3\sin^2 \theta$

$\left\{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

34) $\cos 2\theta + 2 + 3\cos \theta = 0$

$\left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$