Warm-Up

<u>Directions</u>: COMPLETELY simplify in #1 & 2.

1) $\left(y^{2} \cdot x^{\frac{5}{3}} y^{2}\right)^{-\frac{1}{3}}$

2) $\dfrac{y^{-\frac{1}{2}}}{\left(x^{-\frac{1}{4}} y^{\frac{3}{2}}\right)^{2}}$

3)

$$h(x) = \frac{1}{(x-5)^2 + 4(x-5) + 4}$$

For what value of x is the function h above undefined?

Answer key

1) $\left(y^{2} \cdot x^{\frac{5}{3}} y^{2}\right)^{-\frac{1}{3}}$ $\quad \dfrac{y^{\frac{2}{3}} x^{\frac{4}{9}}}{y^{2} x}$

2) $\dfrac{y^{-\frac{1}{2}}}{\left(x^{-\frac{1}{4}} y^{\frac{3}{2}}\right)^{2}}$ $\quad \dfrac{x^{\frac{1}{2}} y^{\frac{1}{2}}}{y^{4}}$

The correct answer is 3. The function $h(x)$ is undefined when the denominator of $\dfrac{1}{(x-5)^2+4(x-5)+4}$ is equal to zero. The expression $(x-5)^2+4(x-5)+4$ is a perfect square: $(x-5)^2+4(x-5)+4 = ((x-5)+2)^2$, which can be rewritten as $(x-3)^2$. The expression $(x-3)^2$ is equal to zero if and only if $x = 3$. Therefore, the value of x for which $h(x$ is undefined is 3.

5-5
Logarithmic Functions

Learning Targets:

- I can define and apply logarithms.

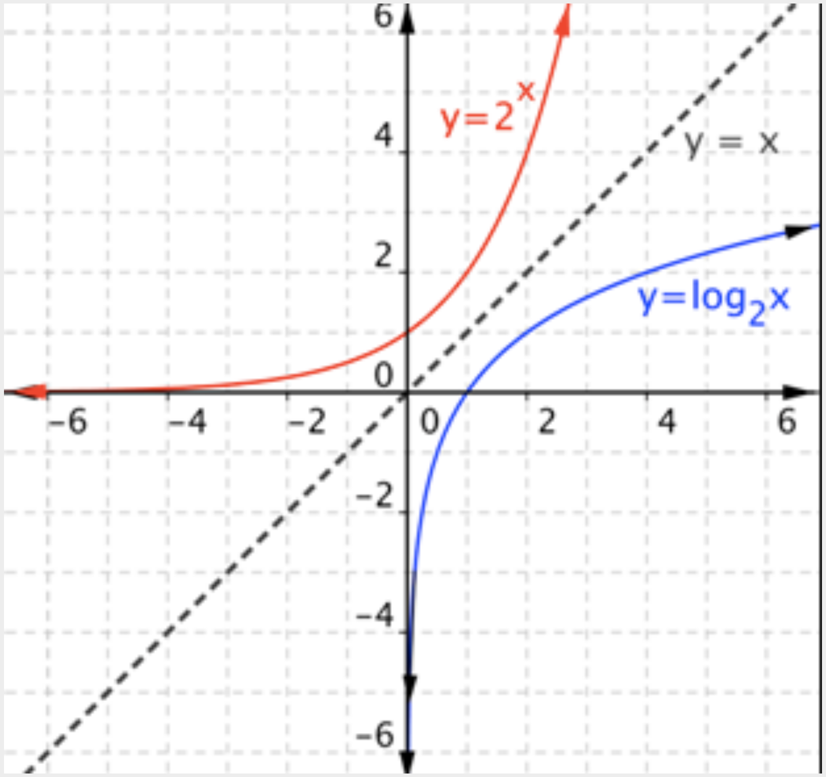
Logarithmic Functions

The inverse of an exponential function is called a *logarithmic function*.

$y = \log_a x$ if and only if $x = a^y$

$$y = \log_a x$$

is called the logarithmic function of x with base a.



Log form to Exponential form…and back

log form	exponential form
$\log_8 w = 6$	$8^6 = w$
$\log_x b = 27$	$x^{27} = b$
$\log_2 d = -9$	$2^{-9} = d$
$\log_{3.14} y = 12.4$	$3.14^{12.4} = y$

You try…

Honors PreCalculus

Name________________________________

Writing in Exponential & Log form

Date_______________ Period____

Rewrite each equation in exponential form.

1) $\log_{13} n = 8$

2) $\log_{16} 256 = 2$

3) $\log_{7} 49 = 2$

4) $\log_{v} 38 = u$

5) $\log_{16} \frac{1}{256} = -2$

6) $\log_{3} 9 = 2$

Rewrite each equation in logarithmic form.

7) $14^{2} = 196$

8) $9^{2} = 81$

9) $12^{x} = y$

10) $v^{3} = u$

11) $7^{n} = m$

12) $b^{a} = 176$

Rewrite each equation in exponential form.

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$13^8 = n$

2) $\log_{16} 256 = 2$

$16^2 = 256$

3) $\log_7 49 = 2$

$7^2 = 49$

4) $\log_v 38 = u$

$v^u = 38$

5) $\log_{16} \frac{1}{256} = -2$

$16^{-2} = \frac{1}{256}$

6) $\log_3 9 = 2$

$3^2 = 9$

Rewrite each equation in logarithmic form.

7) $14^2 = 196$

$\log_{14} 196 = 2$

8) $9^2 = 81$

$\log_9 81 = 2$

9) $12^x = y$

$\log_{12} y = x$

10) $v^3 = u$

$\log_v u = 3$

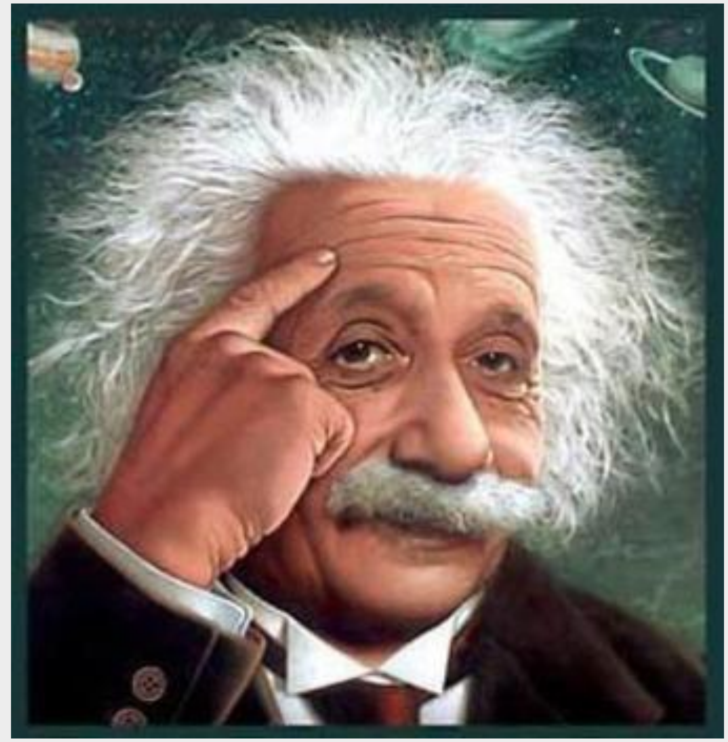
11) $7^n = m$

$\log_7 m = n$

12) $b^a = 176$

$\log_b 176 = a$

Do some logs in your head



1) $\log_5 25 = 2 \longrightarrow 5^2 = 25$

2) $\log_5 125 = 3 \longrightarrow 5^3 = 125$

3) $\log_2 \frac{1}{8} = -3 \longrightarrow 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Common Logs

Logs with a base of 10 are called **common logs**.

The log button on your calculator automatically has a base of 10.

Because the base 10 is the common log, the 10 is often not even written.

Examples:

$$\log_{10} x \to \log x$$

$$\log_{10} 5 \to \log 5$$

$$\log_{10} \frac{1}{3} \to \log \frac{1}{3}$$

Note

$\log 0$ does not exist

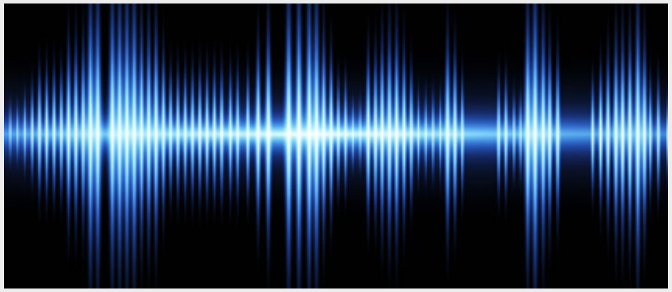
$\log -\#$ does not exist

$\log 0 = y \longrightarrow 10^{y} = 0$

$\log -1 = y \longrightarrow 10^{y} = -1$

Decibel Levels

Common logs are useful in applications involving the perception of sound. Every sound has an intensity level due to the power of the sound wave.



https://www.youtube.com/watch?v=XLfQpv2ZRPU

I_0 represents the intensity of a sound barely audible. The intensity level I of any other sound is measured in terms of I_0.

The unit for measuring the loudness of a sound is the decibel (dB) and it is related to the intensity of a sound by:

$$\text{decibel level of } I = 10 \log \frac{I}{I_0}$$

Sound	I	dB
Barely audible	I_0	0
Whisper	$10I_0$	10
Leaves in a breeze	$10^2 I_0$	20
Soft recorded music	$10^4 I_0$	40
2-person conversation	$10^6 I_0$	60
Loud stereo set	$10^8 I_0$	80
Subway train	$10^{10} I_0$	100
Jet at takeoff	$10^{12} I_0$	120
Pain in eardrum	$10^{13} I_0$	130

Here, the intensity level of each sound is measured in terms of I_0.

$$I = 10\log\frac{I}{I_0}$$

$$= 10\log\frac{10^8 I_0}{I_0}$$

$$= 10\log 10^8$$

$$= 80$$

Many people think that when the intensity of a sound is doubled, the decibel level is doubled also. The following example shows that this is not so.

Example 1: Two loud stereos are playing the same music simultaneously at 80 dB each. What is the decibel level of the combined sound? By how many decibels is the decibel level of the two stereos greater than the decibel level of one stereo?

Example 1: Two loud stereos are playing the same music simultaneously at 80 dB each. What is the decibel level of the combined sound? By how many decibels is the decibel level of the two stereos greater than the decibel level of one stereo?

Since one stereo at 80 dB has an intensity $10^8 I_0$, two stereos will have an intensity that is twice that amount: $2(10^8 I_0)$.

So, the decibel level of 2 stereos is: $I = 10\log\frac{I}{I_0}$

$$= 10\log\frac{2(10^8 I_0)}{I_0}$$

$$\approx 83$$

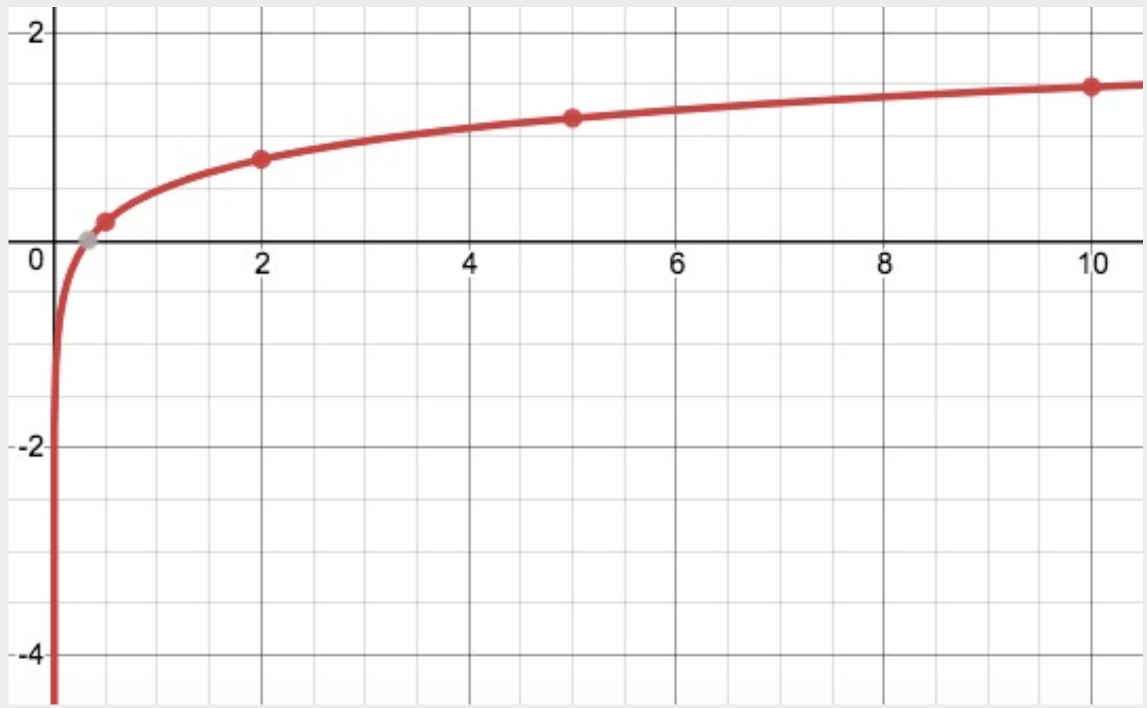
So there is only about a 3 decibel level increase when 2 stereos are played at the same time.

Graphs of Log Functions

You can make a table to graph a log function.

$$f(x) = \log 3x$$

x	y
0.5	0.176
2	0.778
5	1.176
10	0.176



Natural Logs

Logs with a base of e are called **natural logs**.

The **ln** button on your calculator is the log with base e.

Instead of writing a log with a base of e, we write "ln".

Examples:

$$\log_e x \to \ln x$$

$$\log_e 5 \to \ln 5$$

$$\log_e \frac{1}{3} \to \ln \frac{1}{3}$$

Exponential Function:

$$g^{-1}(x) = f(x) = b^x$$

Domain: $\mathbb{R}$ (all real numbers)

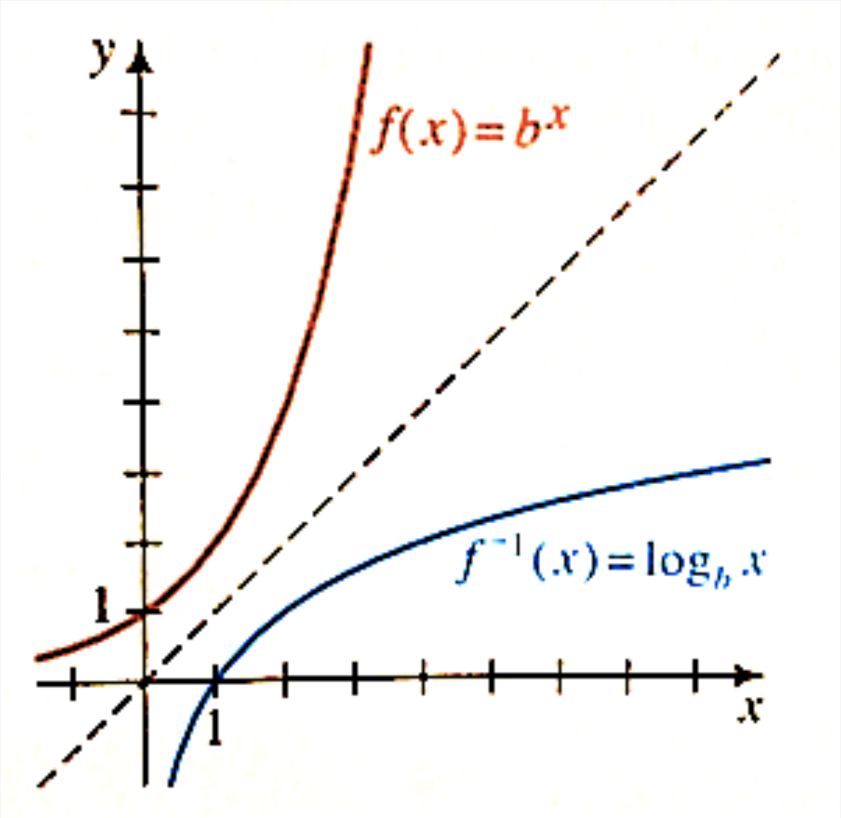
Range: $y > 0$

Logarithmic Function:

$$f^{-1}(x) = g(x) = \log_b x$$

Domain: $x > 0$

Range: $\mathbb{R}$



Practice Problems:

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Written Exercises:

#1-8, 11-14, 17, 18, 24, 25, 31