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The Number e and the Function e^x

Learning Targets:

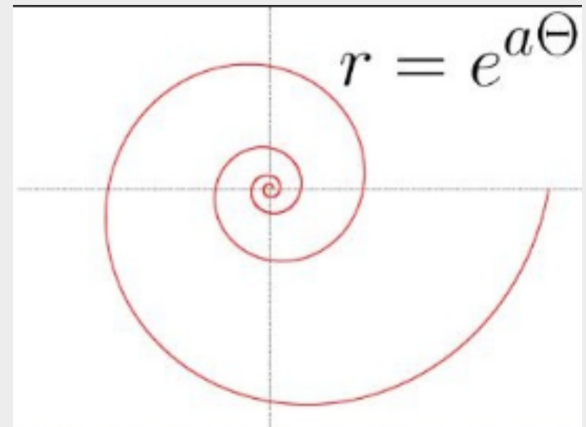
- I can define and apply the natural exponential function.

The number *e*

The number e is an irrational number.

e is also known as Euler's number.

2.718281828459045235360287471352662497757247093699959574966967627724
076630353547594571382178525166427427466391932003059921817413596629
043572900334295260595630738132328627943490763233829880753195251019
011573834187930702154089149934884167509244761460668082264800168477
411853742345442437107539077744992069551702761838606261331384583000
752044933826560297606737113200709328709127443747047230696977209310
141692836819025515108657463772111252389784425056953696770785449969
967946864454905987931636889230098793127736178215424999229576351482
208269895193668033182528869398496465105820939239829488793320362509
443117301238197068416140397019837679320683282376464804295311802328
782509819455815301756717361332069811250996181881593041690351598888
519345807273866738589422879228499892086805825749279610484198444363
463244968487560233624827041978623209002160990235304369941849146314
0934317381436405462531520961836908887070167683964243781405927145



The number *e*

e is defined as:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}$$

which is read as "the limit of $\left(1+\frac{1}{n}\right)^{n}$ as n approaches infinity".

The number e

n	$\left(1+\frac{1}{n}\right)^n$
10	2.59374246
100	2.70481383
1000	2.71692393
10,000	2.71814593
100,000	2.718262824

As the value of n increases, $\left(1+\frac{1}{n}\right)^n$ appears to get closer and closer to 2.718…

The function $y = e^x$ is called the natural exponential function.

Compound Interest and the number $\boldsymbol{e}$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P: principle (initial amount)

r: interest rate (in decimal form)

n: number of times account is being compounded per year

t: number of years

Example:

A total of $12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years if it is compounded:

a) quarterly (4 times per year)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 12{,}000\left(1 + \frac{0.09}{4}\right)^{4\times 5}$$

$$\approx 18{,}726.11$$

Example:

A total of $12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years if it is compounded:

b) monthly (12 times per year)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 12{,}000\left(1 + \frac{0.09}{12}\right)^{12\times 5}$$

$$\approx 18{,}788.17$$

Compound Interest and the number *e*

$$A = Pe^{rt}$$

For continuous compounding, we use a similar formula with the number e.

P: principle (initial amount)

r: interest rate (in decimal form)

t: number of years

Example:

A total of \$12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years if it is compounded:

c) continuously (use the formula with e)

$$A = Pe^{rt}$$

$$A = 12{,}000 \times e^{0.09 \times 5}$$

$$\approx 18{,}819.75$$

Practice Problems:

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