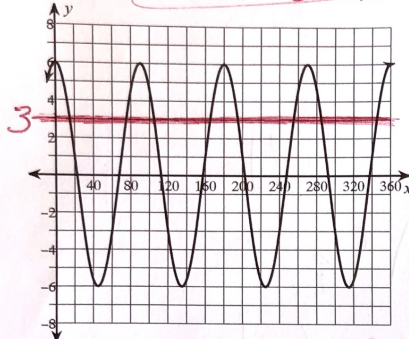


Equations With Period Changes - NOTES

Solve each equation for $0 \leq \theta < 360$.

1) $6 \cos 4\theta = 3$ PERIOD = 90°



2 SOLUTIONS PER PERIOD, 4 PERIODS PER 360° , SO 8 TOTAL ANSWERS.

$$6 \cos 4\theta = 3$$

$$\cos 4\theta = \frac{1}{2}$$

$$4\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$4\theta = 60^\circ$$

$$\theta = 15^\circ$$

$$90 - 15 = 75^\circ$$

$$15 + 90 = 105^\circ, 195^\circ, 285^\circ, 375^\circ$$

$$75 + 90 = 165^\circ, 255^\circ, 345^\circ, 435^\circ$$

3) $-4 + \cos -2\theta = -\frac{7}{2}$ PER = -180°

$$\cos -2\theta = \frac{1}{2}$$

$$-2\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$-2\theta = 60^\circ$$

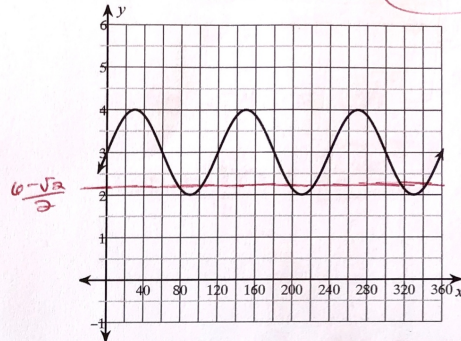
$$\theta = -30^\circ \rightarrow 150^\circ$$

$$-180^\circ - 30^\circ = -210^\circ \rightarrow 150^\circ$$

$$150 + 180 = 330^\circ$$

$$30 + 180 = 210^\circ$$

2) $3 + \sin 3\theta = \frac{6 - \sqrt{2}}{2} \approx 2.29$ PERIOD = 120°



2 SOLUTIONS PER PERIOD, 3 PERIODS PER 360° , SO 6 TOTAL ANSWERS.

$$3 + \sin 3\theta = \frac{6 - \sqrt{2}}{2}$$

$$\sin 3\theta = \frac{-\sqrt{2}}{2}$$

$$3\theta = \sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$$

$$3\theta = -45^\circ$$

$$\theta = -15^\circ \rightarrow 105^\circ$$

$$60 - 15 = 75^\circ$$

$$105 + 120 = 225^\circ, 345^\circ, 465^\circ$$

$$75 + 120 = 195^\circ, 315^\circ, 435^\circ$$

4) $-2 + \cos \frac{\theta}{2} = \frac{-4 - \sqrt{2}}{2}$ PER = 720°

$$\cos \frac{\theta}{2} = \frac{-\sqrt{2}}{2}$$

$$\frac{\theta}{2} = \cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$$

$$\frac{\theta}{2} = 135^\circ$$

$$\theta = 270^\circ$$

$$5) 3 + \sin \frac{\theta}{4} = \frac{7}{2}$$

$$\text{Per} = 1440^\circ$$

$$\sin \frac{\theta}{4} = \frac{1}{2}$$

$$\frac{\theta}{4} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\frac{\theta}{4} = 30^\circ$$

$$\theta = 120^\circ$$

$$720 - 120 = 600^\circ$$

$$6) -2 + \sin 2\theta = -3$$

$$\text{Per} = 180^\circ$$

$$\sin 2\theta = -1$$

$$2\theta = \sin^{-1}(-1)$$

$$2\theta = -90^\circ$$

$$\theta = -45^\circ + 180^\circ \rightarrow 135^\circ$$

$$90 - (-45) = 135^\circ$$

$$135 + 180 = 315^\circ$$

Solve each equation for $0 \leq \theta < 2\pi$.

$$7) -2 + \cos 3\theta = \frac{-4 + \sqrt{2}}{2}$$

$$\text{Per} = \frac{2\pi}{3}$$

$$\cos 3\theta = \frac{\sqrt{2}}{2}$$

$$3\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$3\theta = \frac{\pi}{4}$$

$$\theta = \frac{\pi}{12}$$

$$\frac{2\pi}{3} - \frac{\pi}{12} \rightarrow \frac{8\pi}{12} - \frac{\pi}{12} = \frac{7\pi}{12}$$

$$\frac{\pi}{12} + \frac{2\pi}{3} \rightarrow \frac{\pi}{12} + \frac{8\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

$$\frac{9\pi}{12} + \frac{8\pi}{12} = \frac{17\pi}{12}$$

$$\frac{17\pi}{12} + \frac{8\pi}{12} = \frac{25\pi}{12}$$

$$\frac{7\pi}{12} + \frac{2\pi}{3} \rightarrow \frac{7\pi}{12} + \frac{8\pi}{12} = \frac{15\pi}{12} = \frac{5\pi}{4}$$

$$\frac{15\pi}{12} + \frac{8\pi}{12} = \frac{23\pi}{12}$$

$$8) -1 + \sin 4\theta = \frac{-2 - \sqrt{3}}{2}$$

$$\text{Per} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\sin 4\theta = \frac{-\sqrt{3}}{2}$$

$$4\theta = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

$$4\theta = -\frac{\pi}{3}$$

$$\theta = -\frac{\pi}{12} + \frac{\pi}{2} \rightarrow \frac{5\pi}{12}$$

$$\frac{\pi}{4} - \frac{5\pi}{12} \rightarrow \frac{3\pi}{12} - \frac{5\pi}{12} = \frac{-2\pi}{12} \rightarrow \frac{-\pi}{6} \rightarrow \frac{11\pi}{12}$$

$$\frac{5\pi}{12} + \frac{\pi}{2} \rightarrow \frac{5\pi}{12} + \frac{6\pi}{12} = \frac{11\pi}{12}$$

$$\frac{11\pi}{12} + \frac{6\pi}{12} = \frac{17\pi}{12}$$

$$\frac{17\pi}{12} + \frac{6\pi}{12} = \frac{23\pi}{12}$$

$$\frac{\pi}{3} + \frac{\pi}{2} \rightarrow \frac{2\pi}{6} + \frac{3\pi}{6} = \frac{5\pi}{6}$$

$$\frac{5\pi}{6} + \frac{3\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$$

$$\frac{8\pi}{6} + \frac{3\pi}{6} = \frac{11\pi}{6}$$

$$9) \frac{-2 + \sqrt{3}}{2} = -1 + \cos -3\theta \quad \text{PER} = -\frac{2\pi}{3}$$

$$\frac{\sqrt{3}}{2} = \cos -3\theta$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = -3\theta$$

$$\frac{\pi}{6} = -3\theta$$

$$-\frac{\pi}{18} = \theta \rightarrow \boxed{\frac{11\pi}{18}}$$

$$-\frac{2\pi}{3} - \frac{\pi}{18} \rightarrow \frac{-12\pi}{18} - \frac{\pi}{18} = \frac{-13\pi}{18} \rightarrow \boxed{\frac{5\pi}{18}}$$

$$\frac{11\pi}{18} + \frac{2\pi}{3} \rightarrow \frac{11\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{23\pi}{18}}$$

$$\frac{23\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{35\pi}{18}}$$

$$\frac{\pi}{18} + \frac{2\pi}{3} \rightarrow \frac{\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{13\pi}{18}}$$

$$\frac{13\pi}{18} + \frac{12\pi}{18} = \boxed{\frac{25\pi}{18}}$$

$$11) \frac{5}{2} = 2 + \sin 2\theta \quad \text{PER} = \pi$$

$$\frac{1}{2} = \sin 2\theta$$

$$\sin^{-1}\left(\frac{1}{2}\right) = 2\theta$$

$$\frac{\pi}{6} = 2\theta$$

$$\boxed{\frac{\pi}{12} = \theta}$$

$$\frac{\pi}{2} - \frac{\pi}{12} \rightarrow \frac{6\pi}{12} - \frac{\pi}{12} = \boxed{\frac{5\pi}{12}}$$

$$\frac{\pi}{2} + \pi \rightarrow \frac{\pi}{2} + \frac{12\pi}{12} = \boxed{\frac{13\pi}{12}}$$

$$\frac{5\pi}{2} + \pi \rightarrow \frac{5\pi}{2} + \frac{12\pi}{12} = \boxed{\frac{17\pi}{12}}$$

$$10) 2 + \cos -3\theta = 0 \quad \text{PER} = -\frac{2\pi}{3}$$

$$\cos -3\theta = -2$$

$$-3\theta = \cos^{-1}(-2)$$

NO SOLUTION

$$12) \frac{2}{3} \sin 4\theta = \frac{\sqrt{3}}{3} \quad \text{PER} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\sin 4\theta = \frac{\sqrt{3}}{2}$$

$$4\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$4\theta = \frac{\pi}{3}$$

$$\boxed{\theta = \frac{\pi}{12}}$$

$$\frac{\pi}{4} - \frac{\pi}{12} \rightarrow \frac{3\pi}{12} - \frac{\pi}{12} = \frac{2\pi}{12} = \boxed{\frac{\pi}{6}}$$

$$\frac{\pi}{2} + \frac{\pi}{2} \rightarrow \frac{\pi}{2} + \frac{6\pi}{12} = \boxed{\frac{7\pi}{12}}$$

$$\frac{7\pi}{12} + \frac{6\pi}{12} = \boxed{\frac{13\pi}{12}}$$

$$\frac{13\pi}{12} + \frac{6\pi}{12} = \boxed{\frac{19\pi}{12}}$$

$$\frac{\pi}{6} + \frac{\pi}{2} \rightarrow \frac{\pi}{6} + \frac{3\pi}{6} = \frac{4\pi}{6} = \boxed{\frac{2\pi}{3}}$$

$$\frac{4\pi}{6} + \frac{3\pi}{6} = \boxed{\frac{7\pi}{6}}$$

$$\frac{7\pi}{6} + \frac{3\pi}{6} = \frac{10\pi}{6} = \boxed{\frac{5\pi}{3}}$$