

Using Identities Practice Quiz

Use identities to find the value of each expression.

- 1) Find
- $\sin \theta$
- and
- $\tan \theta$

if $\cos \theta = -\frac{4}{5}$ and $\tan \theta < 0$.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta + \left(-\frac{4}{5}\right)^2 = 1$$

$$\sin^2 \theta + \frac{16}{25} = 1$$

$$\sin^2 \theta = \frac{9}{25}$$

$$\boxed{\sin \theta = -\frac{3}{5}}$$

$$\tan \theta = \frac{\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{5} \cdot \frac{-5}{4} = \boxed{-\frac{3}{4}}$$

- 2) Find
- $\csc \theta$
- and
- $\sin \theta$

if $\cos \theta = -\frac{4}{7}$ and $\sin \theta < 0$.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta + \left(-\frac{4}{7}\right)^2 = 1$$

$$\sin^2 \theta + \frac{16}{49} = 1$$

$$\sin^2 \theta = \frac{33}{49}$$

$$\boxed{\sin \theta = -\frac{\sqrt{33}}{7}}$$

$$\csc \theta = -\frac{7}{\sqrt{33}} = \boxed{-\frac{7\sqrt{33}}{33}}$$

- 3) Find
- $\cos \theta$
- and
- $\sin \theta$

if $\cot \theta = -\frac{3}{4}$ and $\cos \theta > 0$.

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$1 + \left(-\frac{3}{4}\right)^2 = \csc^2 \theta$$

$$1 + \frac{9}{16} = \csc^2 \theta$$

$$\frac{25}{16} = \csc^2 \theta$$

$$-\frac{5}{4} = \csc \theta$$

$$\boxed{-\frac{4}{5} = \sin \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$-\frac{3}{4} = \frac{\cos \theta}{-\frac{4}{5}}$$

$$\cos \theta = \frac{-4}{5} \cdot \frac{-3}{4} = \boxed{\frac{3}{5}}$$

- 4) Find
- $\csc \theta$
- and
- $\sin \theta$

if $\cos \theta = -\frac{4}{5}$ and $\csc \theta > 0$.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta + \left(-\frac{4}{5}\right)^2 = 1$$

$$\sin^2 \theta + \frac{16}{25} = 1$$

$$\sin^2 \theta = \frac{9}{25}$$

$$\boxed{\sin \theta = \frac{3}{5}}$$

$$\downarrow$$

$$\boxed{\csc \theta = \frac{5}{3}}$$

Simplify.

5) $\sin \theta \sec \theta \cot \theta$

$$= \frac{\sin \theta}{1} \cdot \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}$$
$$= \boxed{1}$$

6) $1 + \tan^2 \left(\frac{\pi}{2} - \theta \right)$

$$= 1 + \cot^2 \theta$$
$$= \boxed{\csc^2 \theta}$$

7) $\frac{\sec^2(x) - 1}{\sin^2 x}$

$$= \frac{\sec^2 x - 1}{\sin^2 x}$$

$$= \frac{\tan^2 x}{\sin^2 x}$$

$$= \tan^2 x \cdot \frac{1}{\sin^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \boxed{\sec^2 x}$$

8) $\tan^2 x - \tan^2 x \sin^2 x$

$$= \tan^2 x (1 - \sin^2 x)$$

$$= \tan^2 x (\cos^2 x)$$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1}$$

$$= \boxed{\sin^2 x}$$

Verify the Identity.

$$9) \frac{\sin x \cos(-x)}{1 - \cos^2 x} = \cot x \quad \text{GIVEN}$$

$$\frac{\sin x \cos x}{1 - \cos^2 x} = \cot x \quad \text{EVEN/ODD IDENTITY}$$

$$\frac{\sin x \cos x}{\sin^2 x} = \cot x \quad \text{PYTHAGOREAN IDENTITY}$$

$$\frac{\cos x}{\sin x} = \cot x \quad \text{DIVISION}$$

$$\cot x = \cot x \quad \text{QUOTIENT IDENTITY}$$

$$10) \frac{1 - \sin^2 x}{1 + \cot^2 x} = \sin^2 x \cos^2 x \quad \text{GIVEN}$$

$$\frac{\cos^2 x}{1 + \cot^2 x} = \sin^2 x \cos^2 x \quad \text{PYTHAGOREAN IDENTITY}$$

$$\frac{\cos^2 x}{\csc^2 x} = \sin^2 x \cos^2 x \quad \text{PYTHAGOREAN IDENTITY}$$

$$\cos^2 x \cdot \frac{1}{\csc^2 x} = \sin^2 x \cos^2 x \quad \text{DIVISION}$$

$$\cos^2 x = \sin^2 x = \sin^2 x \cos^2 x \quad \text{RECIPROCAL IDENTITY}$$

$$11) \frac{\tan^2 x}{1 + \tan^2 x} = \sin^2 x \quad \text{GIVEN}$$

$$\frac{\tan^2 x}{\sec^2 x} = \sin^2 x \quad \text{PYTHAGOREAN IDENTITY}$$

$$\tan^2 x \cdot \frac{1}{\sec^2 x} = \sin^2 x \quad \text{DIVISION}$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sec^2 x} = \sin^2 x \quad \text{QUOTIENT IDENTITY}$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x \quad \text{RECIPROCAL IDENTITY}$$

$$\sin^2 x = \sin^2 x \quad \text{DIVISION}$$

$$12) \frac{\sec x - 1}{\csc\left(\frac{\pi}{2} - \theta\right) + 1} = \frac{1 - \cos x}{1 + \cos x} \quad \text{GIVEN}$$

$$\frac{\sec x - 1}{\sec x + 1} = \frac{1 - \cos x}{1 + \cos x} \quad \text{COFUNCTION IDENTITY}$$

$$\frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1} = \frac{1 - \cos x}{1 + \cos x} \quad \text{RECIPROCAL IDENTITY}$$

$$\frac{\frac{1}{\cos x} - \frac{\cos x}{\cos x}}{\frac{1}{\cos x} + \frac{\cos x}{\cos x}} = \frac{1 - \cos x}{1 + \cos x} \quad \text{REWRITE "1"}$$

$$\frac{\frac{1 - \cos x}{\cos x}}{\frac{1 + \cos x}{\cos x}} = \frac{1 - \cos x}{1 + \cos x} \quad \text{SUBTRACTION}$$

$$\frac{1 - \cos x}{\cos x} \cdot \frac{\cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x} \quad \text{DIVISION}$$

$$\frac{1 - \cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x} \quad \text{DIVISION}$$