Warm-Up

Simplify each expression.

1) $\dfrac{36}{12x + 32}$

2) $\dfrac{35x}{35x - 15}$

3) $\dfrac{x - 2}{9x^2 - 18x}$

4) $\dfrac{27m - 90}{18m - 45}$

5) $\dfrac{r^2 - 14r + 48}{r^2 - 2r - 24}$

6) $\dfrac{p - 7}{7p^2 - 50p + 7}$

Warm-Up

Simplify each expression.

1) $\frac{36}{12x + 32}$

$\frac{9}{3x + 8}$

2) $\frac{35x}{35x - 15}$

$\frac{7x}{7x - 3}$

3) $\frac{x - 2}{9x^2 - 18x}$

$\frac{1}{9x}$

4) $\frac{27m - 90}{18m - 45}$

$\frac{3m - 10}{2m - 5}$

5) $\frac{r^2 - 14r + 48}{r^2 - 2r - 24}$

$\frac{r - 8}{r + 4}$

6) $\frac{p - 7}{7p^2 - 50p + 7}$

$\frac{1}{7p - 1}$

13-4: Limits of Infinite Sequences

Learning Targets:

- I can find or estimate the limit of an infinite sequence or determine that the limit does not exist

A series that does not have a last term is called *infinite*.

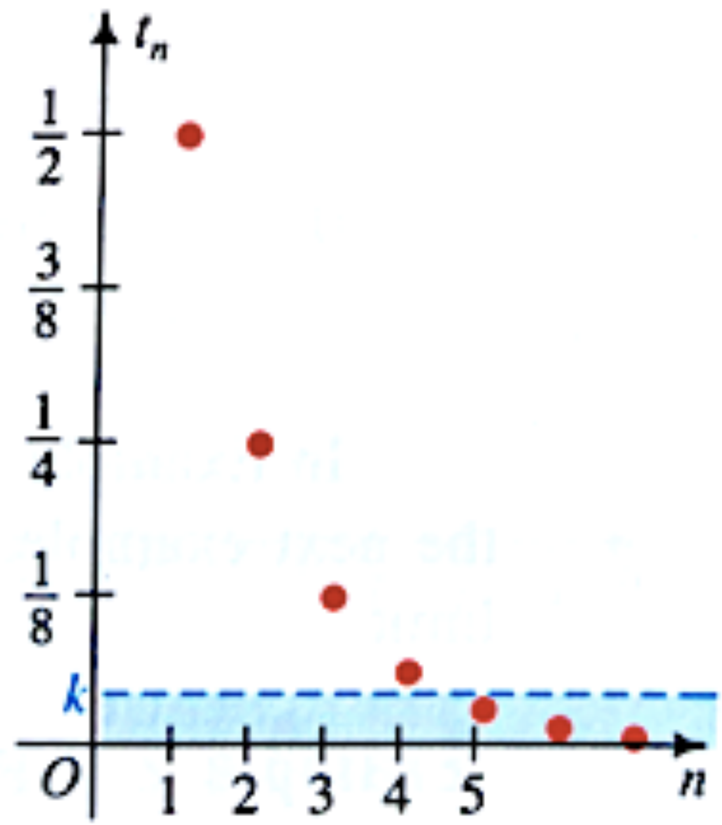
Consider the infinite geometric sequence: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \left(\frac{1}{2}\right)^n, \ldots$

When you substitute larger and larger values of n, $t_n = \left(\frac{1}{2}\right)^n$ becomes a smaller and smaller positive number.

The value of t_n will never become zero.

The graph here illustrates this idea of getting smaller but never reaching 0.

Since the values of $t_n = \left(\frac{1}{2}\right)^n$ get closer to 0 as the values of n get larger, we can say that the sequence approaches 0 as n approaches ∞.



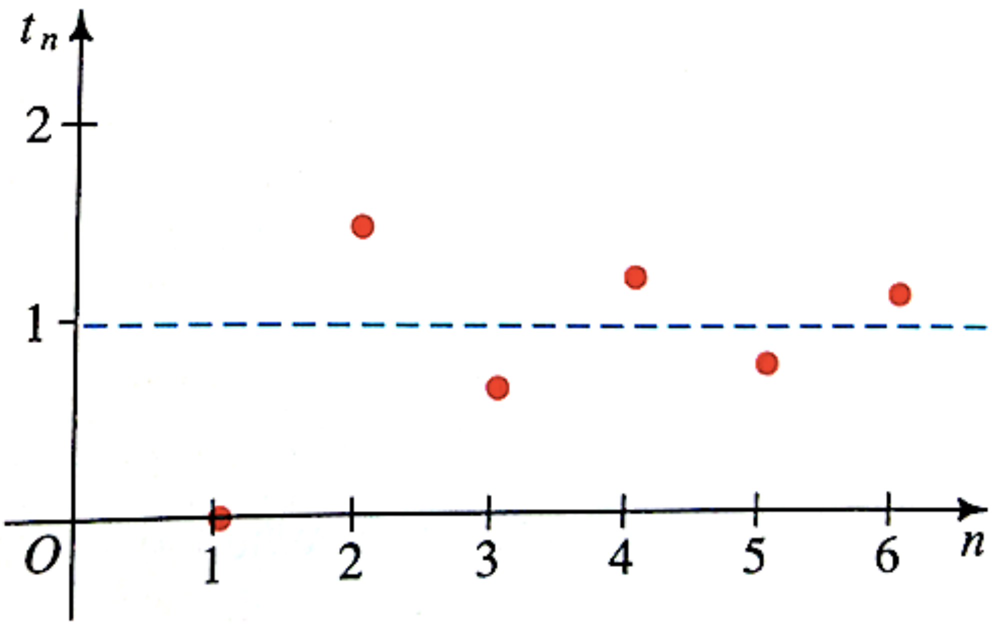
Or, we can write it as:

$$\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$$

This is read, "the limit of $\left(\frac{1}{2}\right)^n$ as n goes to infinity is 0 .

Consider the sequence: $1-\frac{1}{1}, 1+\frac{1}{2}, 1-\frac{1}{3}, 1+\frac{1}{4}, \ldots, 1+\frac{(-1)^n}{n}, \ldots$

If we graph it, we get



$$\lim_{n\to\infty} 1+\frac{(-1)^n}{n} = 1$$

Examples

1) $\lim_{n\to\infty}(0.99)^n$

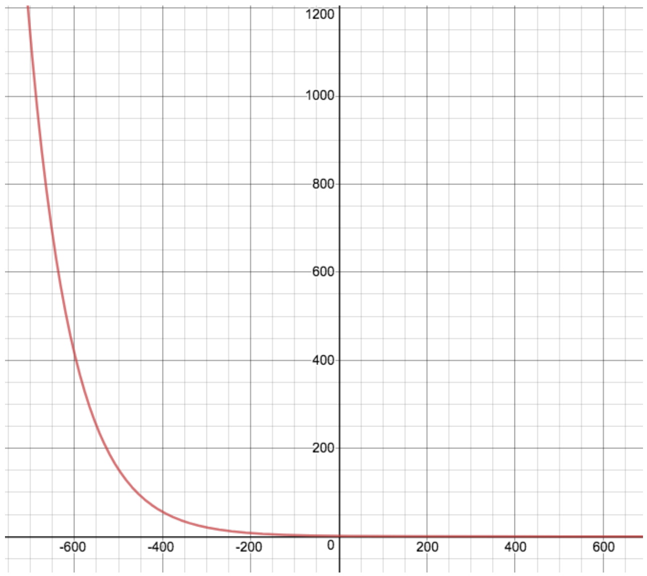
Let's plug in values for n…

$(0.99)^{10} \approx 0.904$

$(0.99)^{1000} \approx 0.000043$

$(0.99)^{10,000} \approx 2.2 \times 10^{-44}$

$\lim_{n\to\infty}(0.99)^n = 0$



Examples

2) $\lim_{n\to\infty} \sin\left(\frac{1}{n}\right)$

Let's plug in values for n…

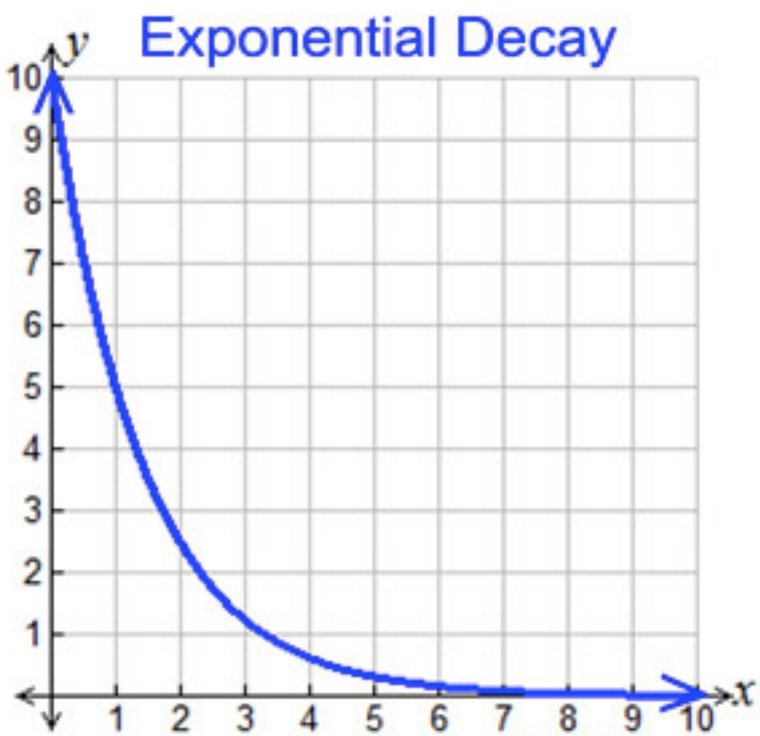
$$\sin\left(\frac{1}{10}\right) \approx 0.0017$$

$$\sin\left(\frac{1}{1000}\right) \approx 0.000017$$

$$\lim_{n\to\infty} \sin\left(\frac{1}{n}\right) = 0$$

Theorem

If $|r| < 1$, then $\lim_{n \to \infty} r^n = 0.$



Examples

3) $\lim_{n\to\infty} \frac{n^2+1}{2n^2-3n}$

Let's simplify a bit by dividing both the numerator and the denominator by the denominator's highest power of n, which in this case is n^2.

$$\frac{n^2+1}{2n^2-3n} \longrightarrow \frac{\frac{n^2+1}{n^2}}{\frac{2n^2-3n}{n^2}} = \frac{\frac{n^2}{n^2}+\frac{1}{n^2}}{\frac{2n^2}{n^2}-\frac{3n}{n^2}} = \frac{1+\frac{1}{n^2}}{2-\frac{3}{n}}$$

As n gets larger, $\frac{1}{n^2}$ and $\frac{3}{n}$ approach 0.

$$\frac{1+0}{2-0} = \frac{1}{2}$$

so, $\lim_{n\to\infty} \frac{n^2+1}{2n^2-3n} = \frac{1}{2}$

Examples

4) $\lim_{n\to\infty} \frac{5n^2+\sqrt{n}}{3n^3+7}$

Again, let's divide both the numerator and the denominator by the denominator's highest power of n, which in this case is n^3.

$$\frac{5n^2+\sqrt{n}}{3n^3+7} \longrightarrow \frac{\frac{5n^2+\sqrt{n}}{n^3}}{\frac{3n^3+7}{n^3}} = \frac{\frac{5n^2}{n^3}+\frac{n^{\frac{1}{2}}}{n^3}}{\frac{3n^3}{n^3}+\frac{7}{n^3}} = \frac{\frac{5}{n}+\frac{1}{n^{\frac{5}{2}}}}{3+\frac{7}{n^3}}$$

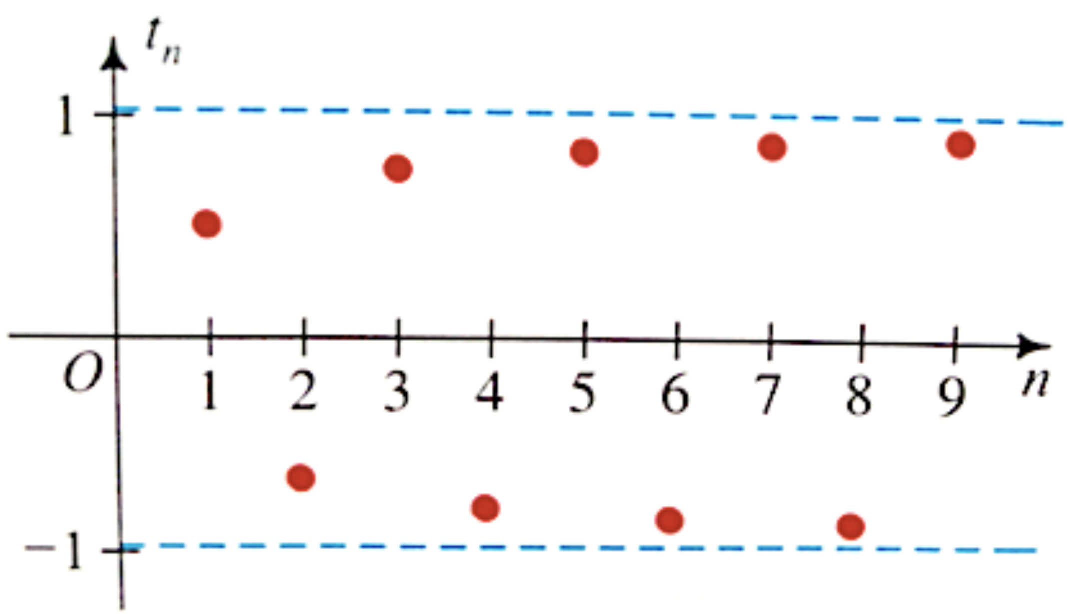
As n gets larger, $\frac{5}{n}$, $\frac{1}{n^{\frac{5}{2}}}$ and $\frac{7}{n^3}$ approach 0.

$$\frac{0+0}{3+0} = \frac{0}{3} = 0$$

so, $\lim_{n\to\infty} \frac{5n^2+\sqrt{n}}{3n^3+7} = 0$

Consider the following sequence:

$$\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \dots, \frac{(-1)^{n+1} * n}{n+1}, \dots$$



As n gets larger, the graph goes towards both 1 and -1.

So,

$$\lim_{n\to\infty} \frac{(-1)^{n+1} * n}{n+1}$$

does not exist.

Consider the following sequence:

$$3, 7, 11, 15, \ldots, 4n - 1, \ldots$$

As n gets larger, the terms in the sequence also get larger.

So,

$$\lim_{n \to \infty} 4n - 1 = \infty$$

Consider the following sequence:

$$-10, -100, -1000, \ldots, -10^n, \ldots$$

As n gets larger, the terms in the sequence get smaller.

So,

$$\lim_{n \to \infty} -10^n = -\infty$$

Practice Problems

Pages 496-497

#1-18