

$$\textcircled{30} \quad \frac{\cot \theta - \tan \theta}{\sin \theta \cos \theta} = \csc^2 \theta - \sec^2 \theta$$

GIVEN

$$\frac{\cot \theta}{\sin \theta \cos \theta} - \frac{\tan \theta}{\sin \theta \cos \theta} = \csc^2 \theta - \sec^2 \theta$$

SUBTRACTION

$$\frac{\frac{\cos \theta}{\sin \theta}}{\sin \theta \cos \theta} - \frac{\frac{\sin \theta}{\cos \theta}}{\sin \theta \cos \theta} = \csc^2 \theta - \sec^2 \theta$$

QUOTIENT IDENTITY

$$\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta \cos \theta} - \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta \cos \theta} = \csc^2 \theta - \sec^2 \theta$$

DIVISION

$$\frac{1}{\sin \theta} \cdot \frac{1}{\sin \theta} - \frac{1}{\cos \theta} \cdot \frac{1}{\cos \theta} = \csc^2 \theta - \sec^2 \theta$$

DIVISION

$$\frac{1}{\sin^2 \theta} - \frac{1}{\cos^2 \theta} = \csc^2 \theta - \sec^2 \theta$$

MULTIPLICATION

$$\csc^2 \theta - \sec^2 \theta = \csc^2 \theta - \sec^2 \theta \quad \square$$

RECIPROCAL IDENTITY

$$\textcircled{31} \quad \frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta} = \sin \theta \csc \theta$$

GIVEN

$$\sin \theta \cdot \frac{1}{\csc \theta} + \cos \theta \cdot \frac{1}{\sec \theta} = \sin \theta \csc \theta$$

DIVISION

$$\sin \theta \cdot \sin \theta + \cos \theta \cdot \cos \theta = \sin \theta \csc \theta$$

RECIPROCAL IDENTITY

$$\sin^2 \theta + \cos^2 \theta = \sin \theta \csc \theta$$

MULTIPLICATION

$$\sin^2 \theta + 1 - \sin^2 \theta = \sin \theta \csc \theta$$

PYTHAGOREAN IDENTITY

$$\sin \theta \left(\sin \theta + \frac{1}{\sin \theta} - \sin \theta \right) = \sin \theta \csc \theta$$

FACTOR

$$\sin \theta \left(\frac{1}{\sin \theta} \right) = \sin \theta \csc \theta$$

SUBTRACTION

$$\sin \theta \csc \theta = \sin \theta \csc \theta \quad \blacksquare$$

RECIPROCAL IDENTITY

$$\textcircled{32} \quad \frac{1 - \sin^2 \theta}{1 + \cot^2 \theta} = \sin^2 \theta \cos^2 \theta$$

GIVEN

$$\frac{\cos^2 \theta}{1 + \cot^2 \theta} = \sin^2 \theta \cos^2 \theta$$

PYTHAGOREAN IDENTITY

$$\frac{\cos^2 \theta}{\csc^2 \theta} = \sin^2 \theta \cos^2 \theta$$

PYTHAGOREAN IDENTITY

$$\frac{1}{\csc^2 \theta} \cdot \cos^2 \theta = \sin^2 \theta \cos^2 \theta$$

DIVISION

$$\sin^2 \theta \cos^2 \theta = \sin^2 \theta \cos^2 \theta \quad \blacksquare$$

RECIPROCAL IDENTITY

$$(33) \quad \tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$$

GIVEN

$$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x}{1} = \tan^2 x \sin^2 x$$

QUOTIENT IDENTITY

$$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x} = \tan^2 x \sin^2 x$$

MULTIPLICATION

$$\frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} = \tan^2 x \sin^2 x$$

SUBTRACTION

$$\frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} = \tan^2 x \sin^2 x$$

FACTOR

$$\frac{\sin^2 x (\sin^2 x)}{\cos^2 x} = \tan^2 x \sin^2 x$$

PYTHAGOREAN IDENTITY

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{\sin^2 x}{1} = \tan^2 x \sin^2 x$$

DIVISION

$$\tan^2 x \cdot \sin^2 x = \tan^2 x \sin^2 x \quad \blacksquare$$

QUOTIENT IDENTITY

$$(34) \quad \frac{\tan^2 x}{1 + \tan^2 x} = \sin^2 x$$

GIVEN

$$\frac{\tan^2 x}{\sec^2 x} = \sin^2 x$$

PYTHAGOREAN IDENTITY

$$\tan^2 x \cdot \frac{1}{\sec^2 x} = \sin^2 x$$

DIVISION

$$\tan^2 x \cdot \cos^2 x = \sin^2 x$$

RECIPROCAL IDENTITY

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x$$

QUOTIENT IDENTITY

$$\sin^2 x = \sin^2 x \quad \blacksquare$$

DIVISION