WRITTEN EXERCISES

Simplify.

A
1. a. $\cos^2\theta + \sin^2\theta$ b. $(1 - \cos\theta)(1 + \cos\theta)$ c. $(\sin\theta - 1)(\sin\theta + 1)$
2. a. $1 + \tan^2\theta$ b. $(\sec x - 1)(\sec x + 1)$ c. $\tan^2 x - \sec^2 x$
3. a. $1 + \cot^2 A$ b. $(\csc A - 1)(\csc A + 1)$ c. $\frac{1}{\sin^2 A} - \frac{1}{\tan^2 A}$
4. a. $\frac{1}{\cos(90° - \theta)}$ b. $1 - \frac{\sin^2\theta}{\tan^2\theta}$ c. $\frac{1}{\cos^2\theta} - \frac{1}{\cot^2\theta}$
5. a. $\cos\theta\cot(90° - \theta)$ b. $\csc^2 x\,(1 - \cos^2 x)$ c. $\cos\theta(\sec\theta - \cos\theta)$
6. a. $\cot A\sec A\sin A$ b. $\cos^2 A\,(\sec^2 A - 1)$ c. $\sin\theta(\csc\theta - \sin\theta)$
7. $\sin A\tan A + \sin(90° - A)$
8. $\csc A - \cos A\cot A$
9. $(\sec B - \tan B)(\sec B + \tan B)$
10. $(1 - \cos B)(\csc B + \cot B)$
11. $(\csc x - \cot x)(\sec x + 1)$
12. $(1 - \cos x)(1 + \sec x)\cos x$

Simplify each expression.

13. $\frac{\sin x\cos x}{1 - \cos^2 x}$
14. $\frac{\tan x + \cot x}{\sec^2 x}$
15. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$
16. $(\sec^2\theta - 1)(\csc^2\theta - 1)$
17. $\frac{\cot^2\theta}{1 + \csc\theta} + \sin\theta\csc\theta$
18. $\frac{\tan^2\theta}{\sec\theta + 1} + 1$
19. $\cos^3 y + \cos y\sin^2 y$
20. $\frac{\sec y + \csc y}{1 + \tan y}$

B
21. $\frac{\cos\theta}{1 + \sin\theta} + \frac{1 + \sin\theta}{\cos\theta}$
22. $\frac{\sin\theta\cot\theta + \cos\theta}{2\tan(90° - \theta)}$
23. $\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta - \cos^2\theta}$
24. $\frac{\sin^2\theta}{1 + \cos\theta}$ (*Hint:* See Exercise 1(b).)
25. Use the equation $\sin^2\theta + \cos^2\theta = 1$ to prove that $\tan^2\theta + 1 = \sec^2\theta$.
26. Use the equation $\sin^2\theta + \cos^2\theta = 1$ to prove that $\cot^2\theta + 1 = \csc^2\theta$.

In Exercises 27 and 28, use a graphing calculator or a computer to graph the given functions.

27. Graph the function $y = \sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x$. What is the domain of this function? What other function could have this same graph? Use trigonometric relationships to verify the suggested identity.
28. Graph the function $y = (\sin x \div \cos x) \div \tan x$. What is the domain of this function? What other function could have this same graph? Use trigonometric relationships to verify the suggested identity.

In Exercises 29–36, prove the given identity.

29. $\cot^2\theta + \cos^2\theta + \sin^2\theta = \csc^2\theta$

30. $\dfrac{\cot\theta - \tan\theta}{\sin\theta\cos\theta} = \csc^2\theta - \sec^2\theta$

31. $\dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta} = \sin\theta\csc\theta$

32. $\dfrac{1-\sin^2\theta}{1+\cot^2\theta} = \sin^2\theta\cos^2\theta$

33. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

34. $\dfrac{\tan^2 x}{1+\tan^2 x} = \sin^2 x$

35. $\dfrac{\sin\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{1+\tan\theta}$

36. $\dfrac{\tan x}{1+\sec x} + \dfrac{1+\sec x}{\tan x} = 2\csc x$

37. $\overline{AB}$ is tangent to the unit circle at $B(1, 0)$.

a. Why is $\triangle OPQ \sim \triangle OAB$?

b. Use part (a) to explain why

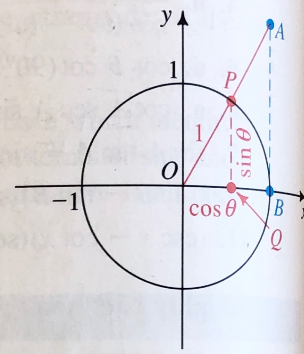
$$\frac{PQ}{OQ} = \frac{AB}{OB} \text{ and } \frac{OP}{OQ} = \frac{AO}{BO}.$$

c. Use part (b) to show that $AB = \tan\theta$ and $AO = \sec\theta$.

d. ***Visual Thinking*** Use the diagram to explain why the name *tangent* is given to the expression $\dfrac{\sin x}{\cos x}$ and the name *secant* is given to the expression $\dfrac{1}{\cos x}$.

e. Use right triangle AOB to prove that $\sec^2\theta = 1 + \tan^2\theta$.

f. Extend $\overline{AO}$ to intersect the circle at C. A theorem from geometry states that $(AB)^2 = AP \cdot AC$. Use this fact to prove $\tan^2\theta = (\sec\theta - 1)(\sec\theta + 1)$.

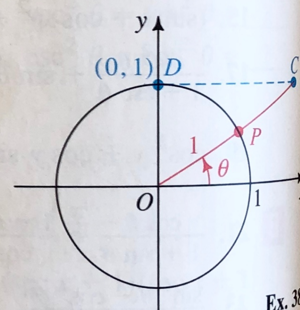


Ex. 37

38. $\overline{CD}$ is tangent to the unit circle at $D(0, 1)$. Show that

$$CD = \cot\theta \text{ and } CO = \csc\theta.$$

(*Hint:* See Exercise 37.)



Ex. 38

39. ***Writing*** Jon expected that the graph of $y = \sqrt{1+\tan^2 x}$ would be the same as the graph of $y = \sec x$, but his graphing calculator showed that this was not the case. Write a paragraph explaining why this happened.

40. Express $\tan\theta$ in terms of $\cos\theta$ only.

41. Express $\sec\theta$ in terms of $\sin\theta$ only.

C **42.** Prove $\sqrt{\dfrac{1-\sin x}{1+\sin x}} = |\sec x - \tan x|$. For what values of x is this identity true?

//// COMPUTER EXERCISE

Imagine that your computer can calculate only the sine function. Write a program for which the input is any x, where $0 \le x \le \frac{\pi}{2}$, and the outputs are the six trigonometric functions of x. (*Hint:* See Exercise 41.)