The Law of Cosines

9-4: Use the law of cosines to find unknown parts of a triangle.

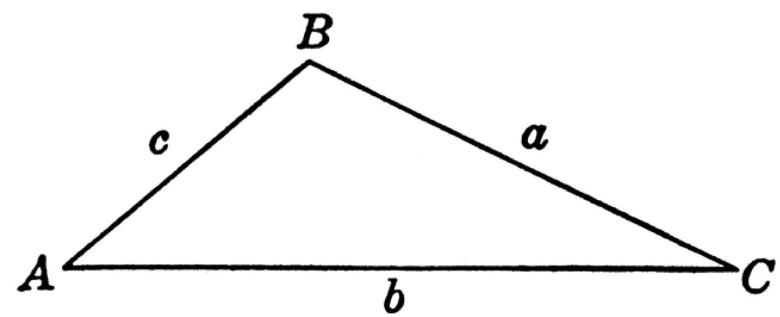
The Law of Cosines

In ΔABC:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

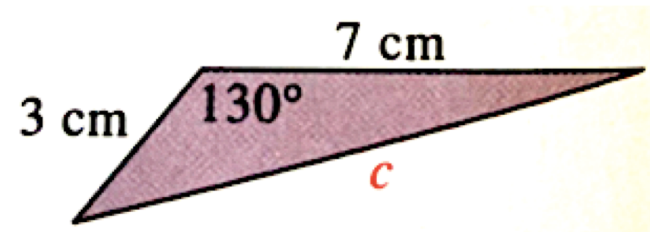
$$a^2 = b^2 + c^2 - 2bc\cos A$$



Example 2:

Suppose that two sides of a triangle have lengths 3 cm and 7 cm and that the angle between them measures 130°. Find the length of the third side.

$$c^2 = a^2 + b^2 - 2ab\cos C$$



$$c^2 = 3^2 + 7^2 - 2 * 3 * 7 * \cos 130^{\circ}$$

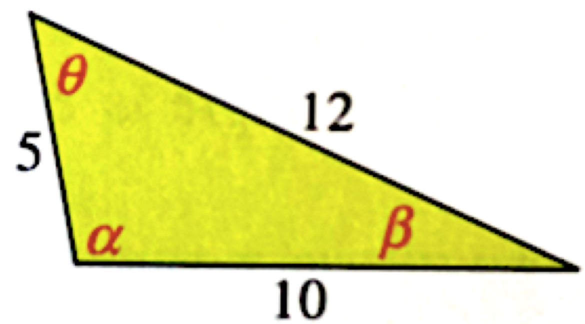
$$c^2 \approx 85$$

$$\boxed{c \approx 9.22\text{cm}}$$

Example 3:

The lengths of the sides of a triangle are 5, 10, and 12. Solve the triangle.

Let's find α first.



$$12^2 = 5^2 + 10^2 - 2 * 5 * 10 * \cos\alpha$$

$$144 = 125 - 100 * \cos\alpha$$

$$19 = -100 * \cos\alpha$$

$$-.19 = \cos\alpha$$

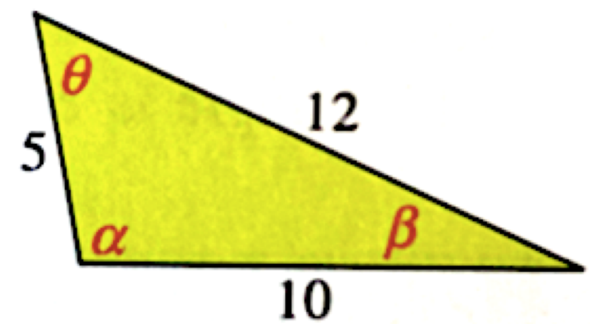
$$\cos^{-1}(-.19) = \alpha$$

$$\boxed{101^\circ \approx \alpha}$$

Example 3:

The lengths of the sides of a triangle are 5, 10, and 12. Solve the triangle.

To find β, we can use either Law of Cosines again or Law of Sines.



$$\frac{\sin\beta}{5} = \frac{\sin\alpha}{12}$$

$$\frac{\sin\beta}{5} = \frac{\sin 101^\circ}{12}$$

$$12\sin\beta = 5\sin 101^\circ$$

$$12\sin\beta = 4.91$$

$$\sin\beta = .41$$

$$\boxed{\beta = 24.14^\circ}$$

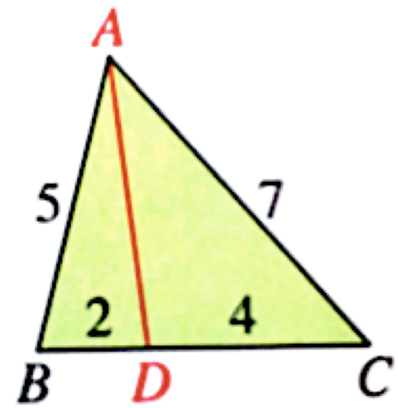
$$\theta = 180^\circ - (101^\circ + 24.14^\circ)$$

$$\boxed{\theta = 54.86^\circ}$$

Example 4:

In the diagram, $AB = 5$, $BD = 2$, $DC = 4$, and $CA = 7$. Find AD.

We can first use the Law of Cosines to find $m\angle B$.



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$7^2 = 6^2 + 5^2 - 2 * 6 * 5 * \cos B$$

$$49 = 61 - 60 * \cos B$$

$$-12 = -60 * \cos B$$

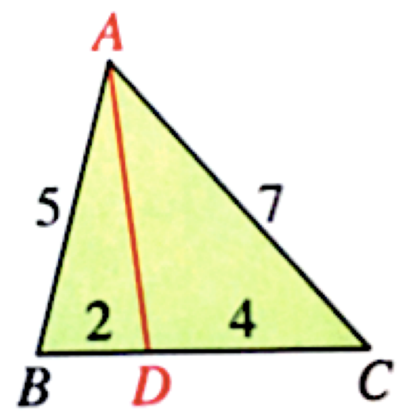
$$0.2 = \cos B$$

$$\cos^{-1}(0.2) = B \longrightarrow \angle B \approx 78.5^{\circ}$$

Example 4:

In the diagram, $AB = 5$, $BD = 2$, $DC = 4$, and $CA = 7$. Find AD.

Now we can use the Law of Cosines again to find AD.



$$b^2 = a^2 + d^2 - 2ad\cos B$$

$$b^2 = 2^2 + 5^2 - 2 * 2 * 5 * \cos 78.5$$

$$b^2 = 25$$

$$b = 5$$

Practice Problems

Pages 352-353 (Written Exercises)

#1-7 odds, 14, 15