Warm-Up

1) Factor $9x^2 - 6x - 8$

$(3x + 2)(3x - 4)$

3) Solve: $\frac{2\pi}{3} = 3\theta$

$\frac{2\pi}{9} = \theta$

2) Factor $10y^2 + 9y + 2$

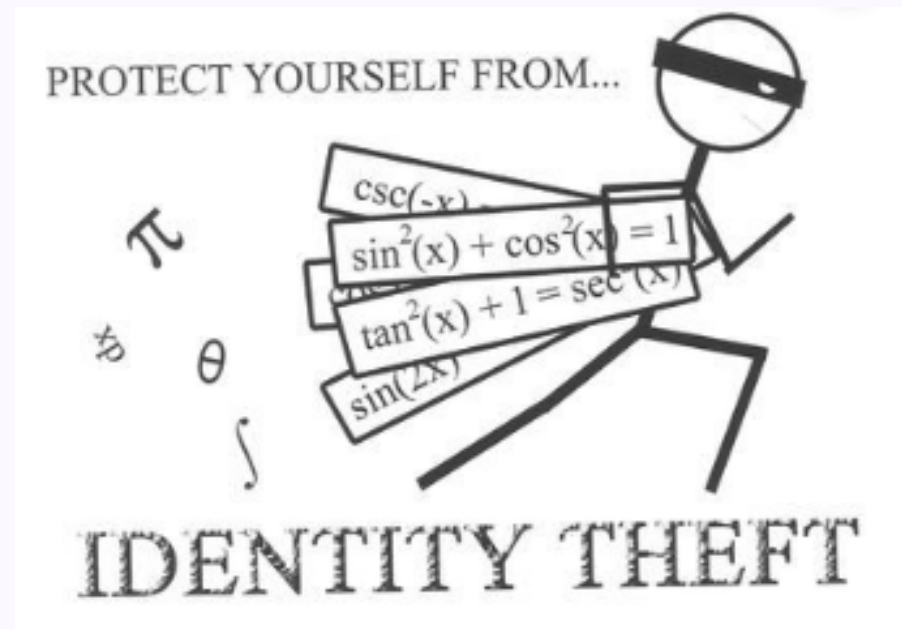$(2y + 1)(5y + 2)$

8-4 Notes: Part 2

Using the Identities

From last time:

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

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IDENTITY THEFT

Pythagorean Identities - Example 1:

Simplify: $\tan^2 \theta - \sec^2 \theta$

$\tan^2 \theta - \sec^2 \theta = \tan^2 \theta - (1 + \tan^2 \theta) = \tan^2 \theta - 1 - \tan^2 \theta = -1$

Pythagorean Identities - Example 2:

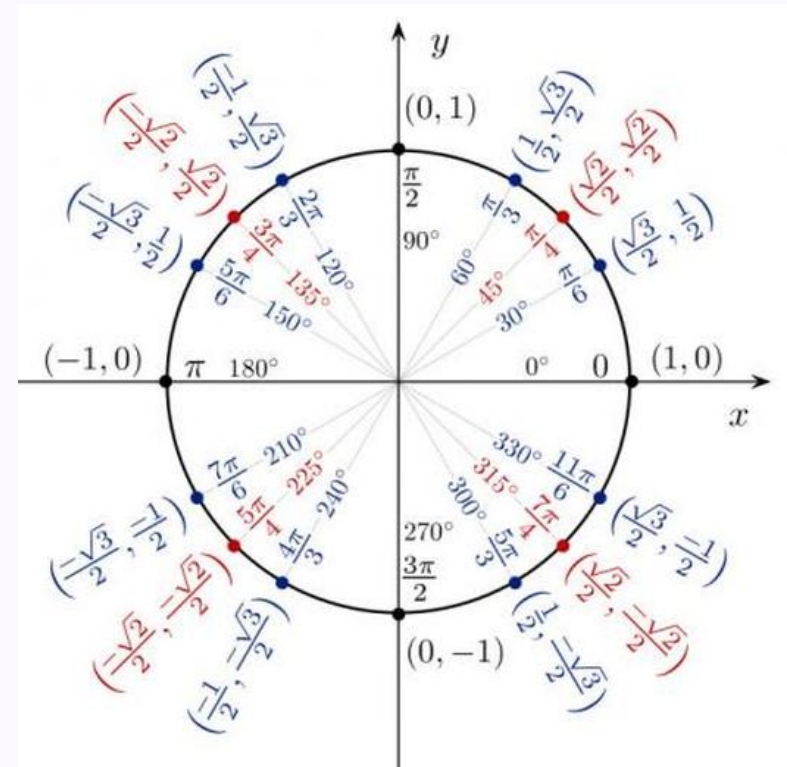Simplify: $\dfrac{\cot A\left(1+\tan^2 A\right)}{\tan A}$

$$\frac{\cot A\left(1+\tan^2 A\right)}{\tan A} = \frac{\cot A \times \sec^2 A}{\tan A} = \frac{1}{\tan A} \times \cot A \times \sec^2 A$$

$$= \cot A \times \cot A \times \sec^2 A = \cot^2 A \times \sec^2 A = \frac{\cos^2 A}{\sin^2 A} \times \frac{1}{\cos^2 A} = \frac{1}{\sin^2 A}$$

$$= \csc^2 A$$

Even/Odd Identities

Trigonometric Functions	
Even Functions $f(-x) = f(x)$	**Odd Functions** $f(-x) = -f(x)$
$\cos(-x) = \cos x$ $\sec(-x) = \sec x$	$\sin(-x) = -\sin x$ $\csc(-x) = -\csc x$ $\tan(-x) = -\tan x$ $\cot(-x) = -\cot x$

Even/Odd Identities - Example 1:

Find all 6 trig functions if:

$$\csc(-x) = -\frac{5}{2} \ and \cot(x) = -\frac{\sqrt{21}}{2}$$

$$\csc(-x) = -\frac{5}{2}$$

csc > 0 and
cot < 0:
Quadrant 2

$$-\csc(x) = -\frac{5}{2}$$

$$\csc(x) = \frac{5}{2}$$

$$\cos x = -\frac{\sqrt{21}}{5} \qquad \sec x = -\frac{5\sqrt{21}}{21}$$

$$\sin x = \frac{2}{5} \qquad \csc x = \frac{5}{2}$$

$$\tan x = -\frac{2\sqrt{21}}{21} \qquad \cot x = -\frac{\sqrt{21}}{2}$$

Even/Odd Identities - Example 2:

Prove: $\sin(-x)\sec(-x) = \tan(-x)$ Given

$$-\sin(x)\sec(-x) = \tan(-x)$$ Even/Odd Identity

$$-\sin(x)\sec(x) = \tan(-x)$$ Even/Odd Identity

$$-\sin(x)\frac{1}{\cos(x)} = \tan(-x)$$ Reciprocal Identity

$$\frac{-\sin(x)}{\cos(x)} = \tan(-x)$$ Multiplication

$$-\tan(x) = \tan(-x)$$ Quotient Identity

$$\tan(-x) = \tan(-x) \ \blacksquare$$ Even/Odd Identity

Cofunction Identities

$$\sin(90° - \theta) = \cos\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\csc(90° - \theta) = \sec\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

$$\cos(90° - \theta) = \sin\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sec(90° - \theta) = \csc\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan(90° - \theta) = \cot\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\cot(90° - \theta) = \tan\theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Cofunction Identities - Example 1:

Find all 6 trig functions if:
$\sin\left(\frac{\pi}{2} - \theta\right) = \frac{4}{7}$, and $\sin(\theta) > 0$.

$\sin > 0$ and $\cos > 0$:
Quadrant 1

$$\cos\theta = \frac{4}{7} \qquad \sec\theta = \frac{7}{4}$$

$$\sin\theta = \frac{\sqrt{33}}{7} \qquad \csc\theta = \frac{7\sqrt{33}}{33}$$

$$\tan\theta = \frac{\sqrt{33}}{4} \qquad \cot\theta = \frac{4\sqrt{33}}{33}$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta = \frac{4}{7}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(\frac{4}{7}\right)^2 = 1$$

$$\sin^2\theta = \frac{33}{49}$$

$$\sin\theta = \frac{\sqrt{33}}{7}$$

$$\tan\theta = \frac{\frac{\sqrt{33}}{7}}{\frac{4}{7}} = \frac{\sqrt{33}}{7} \times \frac{7}{4} = \frac{\sqrt{33}}{4}$$

Cofunction Identities - Example 2:

Prove: $\sin\left(\dfrac{\pi}{2} - x\right)\csc x = \cot x$

$$\sin\left(\dfrac{\pi}{2} - x\right)\csc x = \cot x \qquad \text{Given}$$

$$\cos x \csc x = \cot x \qquad \text{Cofunction Identity}$$

$$\cos x \dfrac{1}{\sin x} = \cot x \qquad \text{Reciprocal Identity}$$

$$\dfrac{\cos x}{\sin x} = \cot x \qquad \text{Multiplication}$$

$$\cot x = \cot x \ \blacksquare \qquad \text{Quotient Identity}$$

Cofunction Identities - Example 3:

Prove: $\dfrac{\frac{1}{\csc x}}{\cos\left(\frac{\pi}{2}-x\right)} + \cot^2\left(\frac{\pi}{2}-x\right) = \sec^2 x$ Given

$\dfrac{\sin x}{\cos\left(\frac{\pi}{2}-x\right)} + \cot^2\left(\frac{\pi}{2}-x\right) = \sec^2 x$ Reciprocal Identity

$\dfrac{\sin x}{\sin x} + \cot^2\left(\frac{\pi}{2}-x\right) = \sec^2 x$ Cofunction Identity

$1 + \tan^2 x = \sec^2 x$ Division & Cofunction Identity

$\sec^2 x = \sec^2 x$ ∎   Pythagorean Identity

Factoring:

a) Factor: $2x^2 + 17x + 8$

$2x^2 + 16x + 1x + 8$

$2x(x + 8) + 1(x + 8)$

$(2x + 1)(x + 8)$

b) Factor: $5m^2 + 36m + 7$

$5m^2 + 35m + 1m + 7$

$5m(m + 7) + 1(m + 7)$

$(5m + 1)(m + 7)$

c) Factor: $5\cos^2\theta + 33\cos\theta - 14$

$5\cos^2\theta + 35\cos\theta - 2\cos\theta - 14$

$5\cos\theta(\cos\theta + 7) - 2(\cos\theta + 7)$

$(5\cos\theta - 2)(\cos\theta + 7)$

d) Factor: $2\sec^2\theta + 5\sec\theta - 25$

$2\sec^2\theta - 5\sec\theta + 10\sec\theta - 25$

$\sec\theta(2\sec\theta - 5) + 5(2\sec\theta - 5)$

$(2\sec\theta - 5)(\sec\theta + 5)$

Practice Problems

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