Honors Precalculus

Name KEY

Using Pythagorean Identities

Date________ Period____

Use identities to find the value of each expression.

1) Find $\sin\theta$ and $\tan\theta$ if $\csc\theta = \frac{7}{4}$ and $\sec\theta > 0$. Q1

$\boxed{\sin\theta = \frac{4}{7}}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{4}{7}\right)^2 + \cos^2\theta = 1$

$\frac{16}{49} + \cos^2\theta = 1$

$\cos^2\theta = \frac{33}{49}$

$\cos\theta = \frac{\sqrt{33}}{7}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{7}}{\frac{\sqrt{33}}{7}} = \frac{4}{7}\cdot\frac{7}{\sqrt{33}}$

$= \frac{4}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$

$\boxed{\tan\theta = \frac{4\sqrt{33}}{33}}$

2) Find $\tan\theta$ and $\csc\theta$ if $\cot\theta = -3$ and $\sin\theta < 0$. Q4

$\frac{\cos\theta}{-\sin\theta}$

$\boxed{\tan\theta = -\frac{1}{3}}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + (-3)^2 = \csc^2\theta$

$10 = \csc^2\theta$

$\boxed{-\sqrt{10} = \csc\theta}$

3) Find $\sec\theta$ and $\csc\theta$ if $\cot\theta = 3$ and $\sin\theta < 0$. Q3

$1 + \cot^2\theta = \csc^2\theta$

$1 + (3)^2 = \csc^2\theta$

$10 = \csc^2\theta$

$\boxed{-\sqrt{10} = \csc\theta}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \left(\frac{1}{3}\right)^2 = \sec^2\theta$

$1 + \frac{1}{9} = \sec^2\theta$

$\frac{10}{9} = \sec^2\theta$

$\boxed{-\frac{\sqrt{10}}{3} = \sec\theta}$

4) Find $\tan\theta$ and $\sec\theta$ if $\csc\theta = -\frac{9}{5}$ and $\cot\theta < 0$. Q4

$1 + \cot^2\theta = \csc^2\theta$

$1 + \cot^2\theta = \left(-\frac{9}{5}\right)^2$

$1 + \cot^2\theta = \frac{81}{25}$

$\cot^2\theta = \frac{56}{25}$

$\cot\theta = -\frac{\sqrt{56}}{5} = -\frac{\sqrt{4\cdot 14}}{5} = \frac{-2\sqrt{14}}{5}$

$\boxed{\cot\theta = \frac{-2\sqrt{14}}{5}}$ $\rightarrow \tan\theta = \frac{-5}{2\sqrt{14}}$

$= \frac{-5\sqrt{14}}{28}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \left(\frac{-5\sqrt{14}}{28}\right)^2 = \sec^2\theta$

$1 + \frac{350}{784} = \sec^2\theta$

$\frac{1134}{784} = \sec^2\theta$

$\frac{\sqrt{81\cdot 14}}{28} = \frac{9\sqrt{14}}{28}$

$\boxed{\sec\theta = \frac{9\sqrt{14}}{28}}$

5) Find csc θ and sin θ if $\tan\theta = \frac{2}{5}$ and $\cos\theta < 0$.

Q3

$\cot\theta = \frac{5}{2}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \left(\frac{5}{2}\right)^2 = \csc^2\theta$

$1 + \frac{25}{4} = \csc^2\theta$

$\frac{29}{4} = \csc^2\theta$

$\boxed{\frac{-\sqrt{29}}{2} = \csc\theta}$

$\sin\theta = \frac{-2}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}$

$\boxed{\sin\theta = \frac{-2\sqrt{29}}{29}}$

6) Find csc θ and sec θ if $\cot\theta = \frac{5}{9}$ and $\cos\theta > 0$.

Q1

$1 + \cot^2\theta = \csc^2\theta$

$1 + \left(\frac{5}{9}\right)^2 = \csc^2\theta$

$1 + \frac{25}{81} = \csc^2\theta$

$\frac{106}{81} = \csc^2\theta$

$\boxed{\frac{\sqrt{106}}{9} = \csc\theta}$

$\tan\theta = \frac{9}{5}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \left(\frac{9}{5}\right)^2 = \sec^2\theta$

$1 + \frac{81}{25} = \sec^2\theta$

$\frac{106}{25} = \sec^2\theta$

$\boxed{\frac{\sqrt{106}}{5} = \sec\theta}$

7) Find csc θ and tan θ if $\sec\theta = -2$ and $\tan\theta > 0$.

Q3

$1 + \tan^2\theta = \sec^2\theta$

$1 + \tan^2\theta = (-2)^2$

$1 + \tan^2\theta = 4$

$\tan^2\theta = 3$

$\boxed{\tan\theta = \sqrt{3}}$

$\cot\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \left(\frac{\sqrt{3}}{3}\right)^2 = \csc^2\theta$

$1 + \frac{3}{9} = \csc^2\theta$

$\frac{12}{9} = \csc^2\theta$

$-\frac{\sqrt{12}}{3} = \csc\theta$

$-\frac{\sqrt{12}}{3} = \frac{-\sqrt{4\cdot 3}}{3} = \boxed{\frac{-2\sqrt{3}}{3} = \csc\theta}$

8) Find tan θ and sin θ if $\cos\theta = -\frac{1}{2}$ and $\tan\theta > 0$.

Q3

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{1}{2}\right)^2 = 1$

$\sin^2\theta + \frac{1}{4} = 1$

$\sin^2\theta = \frac{3}{4}$

$\boxed{\sin\theta = \frac{-\sqrt{3}}{2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}}$

$= \frac{-\sqrt{3}}{2} \cdot \frac{-2}{1} = \sqrt{3}$

$\boxed{\tan\theta = \sqrt{3}}$