WRITTEN EXERCISES

Throughout the exercises, give areas in radical form or to three significant digits. Give lengths to three significant digits and angle measures to the nearest tenth of a degree. In Exercises 1–4, find the area of each $\triangle ABC$.

A

1. a. $a = 4,\ b = 5,\ \angle C = 30°$
b. $a = 4,\ b = 5,\ \angle C = 150°$

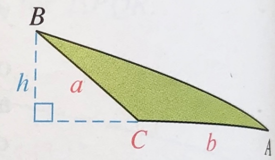
2. a. $b = 3,\ c = 8,\ \angle A = 120°$
b. $b = 3,\ c = 8,\ \angle A = 60°$

3. a. $a = 6,\ c = 2,\ \angle B = 45°$
b. $a = 6,\ c = 2,\ \angle B = 135°$

4. a. $a = 10,\ b = 20,\ \angle C = 70°$
b. $a = 10,\ b = 20,\ \angle C = 110°$

5. What does the formula $K = \frac{1}{2}ab \sin C$ become when $\angle C$ is a right angle? Draw a sketch to illustrate.

6. As shown in the diagram at the right, $\angle C$ in $\triangle ABC$ is obtuse. Show that the formula $K = \frac{1}{2}ab \sin C$ gives the area K of $\triangle ABC$.



Ex. 6

7. Find the area of $\triangle XYZ$ if $x = 16$, $y = 25$, and $\angle Z = 52°$.

8. Find the area of $\triangle RST$ if $\angle S = 125°$, $r = 6$, and $t = 15$.

9. The area of $\triangle ABC$ is 15. If $a = 12$ and $b = 5$, find the measure(s) of $\angle C$.

10. The area of $\triangle PQR$ is 9. If $q = 4$ and $r = 9$, find the measure(s) of $\angle P$.

11. Find the area of a regular octagon inscribed in a circle of radius 40 cm.

12. Find the area of a regular 12-sided polygon inscribed in a circle of radius 8 cm.

13. Adjacent sides of a parallelogram have lengths 6 cm and 7 cm, and the measure of the included angle is 30°. Find the area of the parallelogram.

14. Sketch a parallelogram with sides of lengths a and b and with an acute angle θ. Express the area of the parallelogram in terms of a, b, and θ.

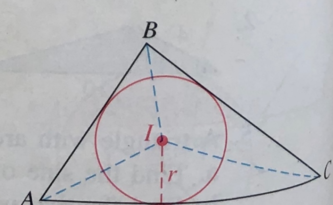
15. Suppose a triangle has two sides of lengths 3 cm and 4 cm and an included angle θ. Express the area of the triangle as a function of θ. State the domain and range of the function and sketch its graph.

16. Suppose a triangle has two sides of lengths a and b. If the angle between these sides varies, what is the maximum possible area that the triangle can attain? What can you say about the minimum possible area?

B

17. a. Given $\triangle ABC$ with an inscribed circle as shown at the right, show that the radius r of the circle is given by:

$$r = \frac{2(\text{area of } \triangle ABC)}{\text{perimeter of } \triangle ABC}$$



(*Hint*: If I is the center of the inscribed circle, then: area of $\triangle ABI$ + area of $\triangle ACI$ + area of $\triangle BCI$ = area of $\triangle ABC$.)

b. Find the radius of the inscribed circle if $AB = AC = 10$ and $BC = 16$.

Part (b) of Exercise 18 requires the use of a computer or graphing calculator. Give your answer to the nearest tenth.

18. In isosceles triangle ABC, $AB = BC = 10$ and $AC = 2x$.

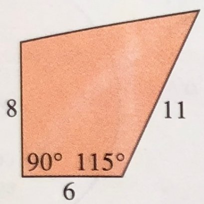
a. Use Exercise 17 to show that the radius of the inscribed circle is:

$$r = \frac{x\sqrt{100 - x^2}}{10 + x}$$

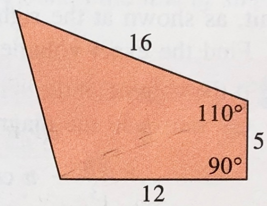
b. Use a computer or graphing calculator to find the value of x that maximizes r. Interpret your answer.

Find the area of each quadrilateral to the nearest square unit.

19.



20.



21. Find the area of a segment of a circle of radius 5 if the measure of the central angle of the segment is 2 radians.

22. Find the area of the segment formed by a chord 24 cm long in a circle of radius 13 cm.

Exercises 23 and 24 require the use of a computer or graphing calculator. Give answers to the nearest tenth.

23. In a circle of radius 10, there is a segment with area 95. Use a computer or graphing calculator to find the measure of the central angle of the segment.

24. For the cylindrical oil tank described in Example 3 on pages 340 and 341, use a computer or graphing calculator to determine where on a measuring rod marks should be put to show that the tank is $\frac{1}{3}$ full and $\frac{2}{3}$ full.

25. ***Visual Thinking*** In Example 3 on pages 340 and 341, suppose the cylindrical oil tank sits upright on one of its circular ends (with the opening for the measuring rod at the other end). Describe how the problem of measuring the amount of oil in the tank changes.

26. ***Research*** Find out and report on what the typical shape of a car's fuel tank is and how the amount of fuel in the tank is measured and indicated on the car's fuel gauge. Is the measuring instrument designed to give a truly accurate measurement of fuel? If not, why not?

Graph the region satisfying both inequalities and find its area.

27. $x^2 + y^2 \leq 36,\ y \geq 3$

28. $x^2 + y^2 \leq 9,\ x \geq 1$

C **29.** $x^2 + y^2 \leq 9,\ x^2 + y^2 - 10x + 9 \leq 0$

30. $x^2 + y^2 - 8y \leq 0,\ x^2 + y^2 \leq 16$