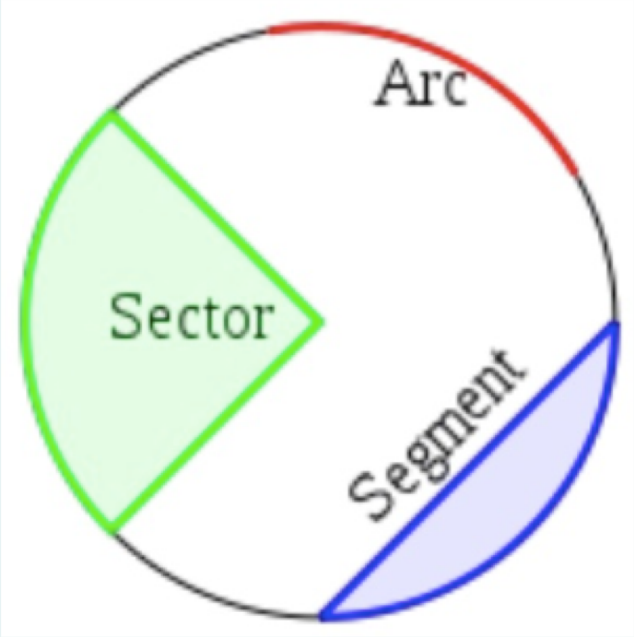
7-2 – Sectors of Circles

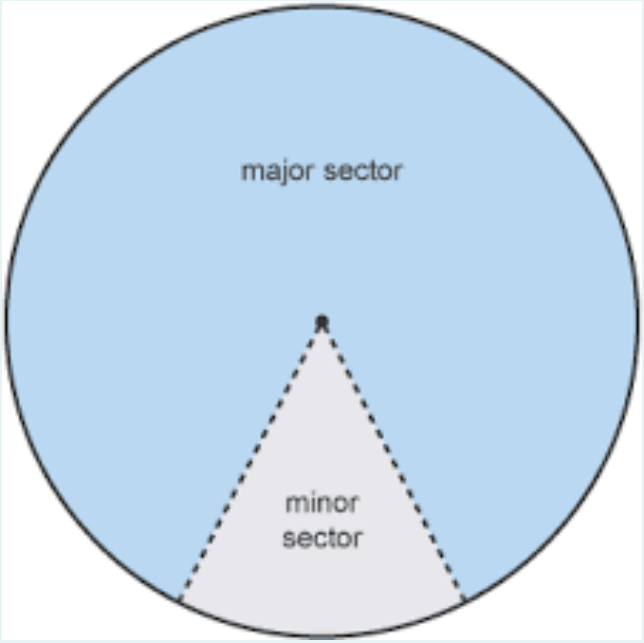
Chapter 7 – Trigonometric Functions

Learning Targets:

- Find the arc length and area of a sector of a circle.
- Solve problems involving apparent size.

A *sector* of a circle is the region bounded by a central angle and the intercepted arc.





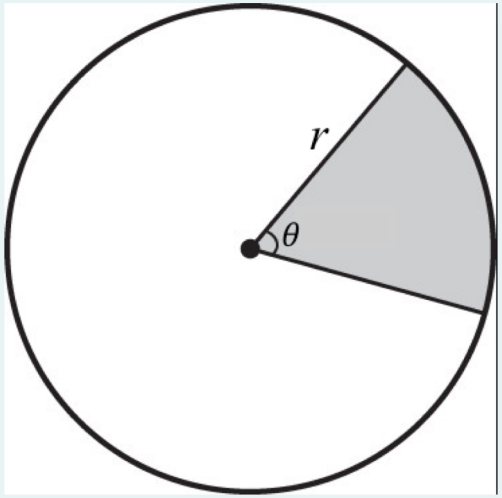
A sector's arc length, s, is a fraction of the circumference.

If θ is in radians, then:

$$s = r\theta$$

If θ is in degrees, then:

$$s = \frac{\pi r \theta}{180°}$$



A sector's area, K, is a fraction of the circle's area.

If θ is in radians, then:

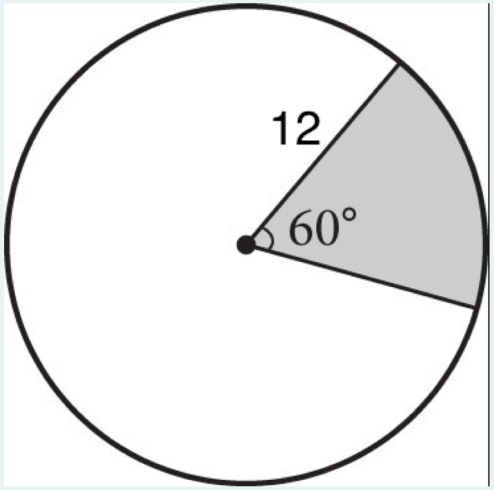
$$K = \frac{1}{2}r^2\theta = \frac{1}{2}rs$$

If θ is in degrees, then:

$$K = \frac{\pi r^2 \theta}{360°}$$

Examples:

1) Find the arc length and area of the sector shown.



Arc Length:

$$s = \frac{\pi r \theta}{180°}$$

$$= \frac{\pi \times 12 \times 60°}{180°}$$

$$= 4\pi \approx 12.6$$

Area:

$$K = \frac{\pi r^2 \theta}{360°}$$

$$= \frac{\pi \times 12^2 \times 60°}{360°}$$

$$= 24\pi \approx 75.4$$

Examples:

1) A sector of a circle has arc length 6 cm and area 75 cm^2. Find its radius and measure of its central angle.

$$K = \frac{1}{2}rs$$

$$75 = \frac{1}{2} \times r \times 6$$

$$25 = r$$

$$s = r\theta$$

$$6 = 25\theta$$

$$0.24 = \theta$$

$$0.24 \text{ radians} \approx 14°$$

Apparent Size

How big an object looks depends on its size and on the angle that it subtends at our eyes. The measure of this angle is called the object's *apparent size*.

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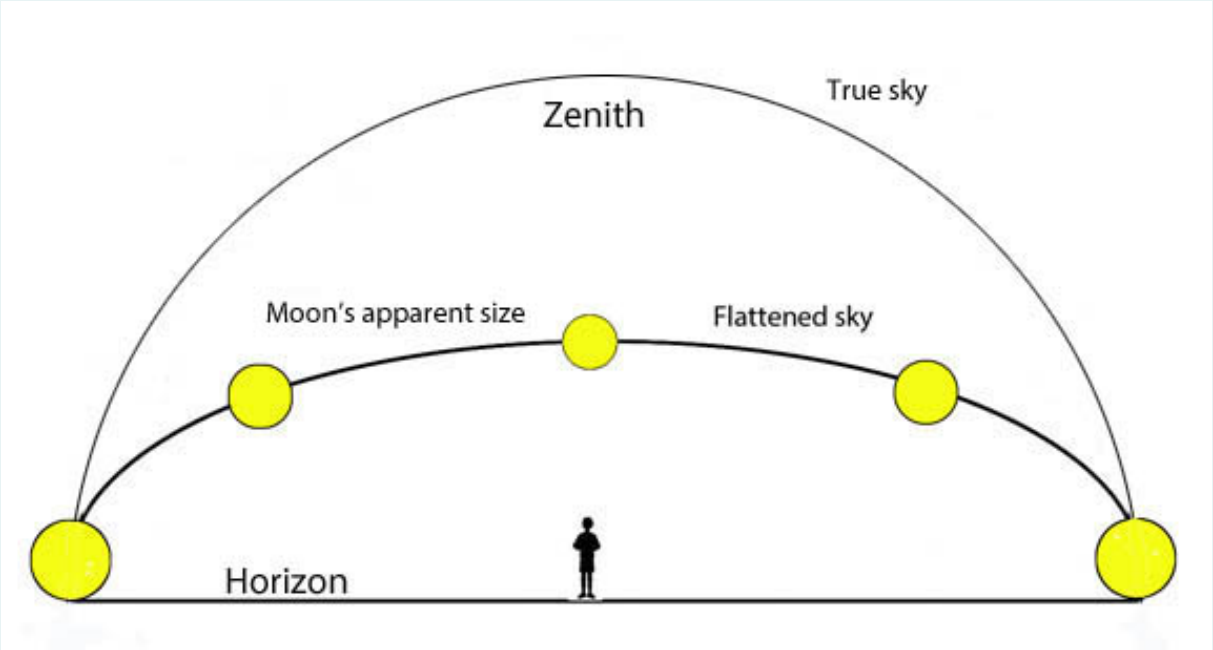
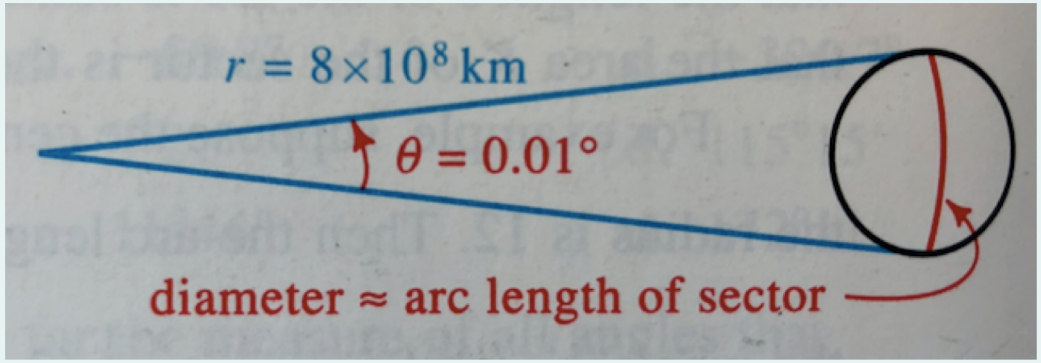


Link explaining moon illusions

Example:

Jupiter has an apparent size of $0.01°$ when it is 8×10^8 km from Earth. Find the approximate diameter of Jupiter.



$$\text{diameter} \approx s \approx \frac{0.01}{360}(2\pi)(8 \times 10^8)$$

$$\approx 140{,}000 \text{ km}$$

Practice Problems:

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