

#### Simplify each expression.

2) 
$$\frac{16n^2 - 48n}{32n}$$
 3)  $\frac{10m + 60}{m + 6}$ 

4) 
$$\frac{p^2 - 13p + 42}{10p - 60}$$



# Unit 5

# **Exponents & Logarithms**

# 5-1: Growth & Decay Integral Exponents

Learning Targets:

• I can define and apply integral exponents

Suppose that the cost of a hamburger has been increasing at the rate of 9% per year. Then, each year the cost is 1.09 times the cost in the previous year.

Suppose that the cost now is \$4. Some projected future costs are given in the table below.

Time (years from now)	0	1	2	3	t
Cost (dollars)	4	4(1.09)	$4(1.09)^2$	$4(1.09)^3$	$4(1.09)^t$

The table suggests that the cost is a function of time t. Since the variable t occurs as an exponent, the cost is said to be an *exponential function* of time:

 $C(t) = 4(1.09)^t$ 

When t > 0, the function gives future costs, and when t < 0, the function gives costs in the past.

### Example 1

Use the cost function  $C(t) = 4(1.09)^t$  to find the cost of a hamburger:

a) 5 years from now  $C(5) = 4(1.09)^5$   $\approx 4(1.54)$   $\approx 6.15$   $\implies$   $\approx $6.15$ b) 5 years ago  $C(5) = 4(1.09)^{-5}$   $\approx 4(0.65)$  $\approx 2.6$   $\implies$   $\approx $2.60$ 



#### Fun fact: The world's most expensive hamburger

Cost: \$5000.00

#### Location: Fleur restaurant, inside Mandalay Bay Casino in Las Vegas





## **Exponential Growth**

 $f(x) = ab^{x}$  $a \neq 0$ b > 1



### **Exponential Decay**

 $f(x) = ab^{x}$  $a \neq 0$ 0 < b < 1



### Example 2

Suppose that a radioactive isotope decays so that the radioactivity present decreases by 15% per day. If 40 kg are present now, find the amount present:



a) 6 days from now

$$A(t) = A_0 (1+r)^t$$
  
 $\approx 40(1-0.15)^6$   
 $\approx 15.1 \text{ kg}$ 

b) 6 days ago

 $\approx 40(1 - 0.15)^{-6}$  $\approx 106.1 \text{ kg}$ 

#### Radioactive Decay: Chernobyl disaster

What happened: Massive explosion at a nuclear power plant.



#### Radioactive Decay: Chernobyl disaster

Where is Chernobyl: Ukraine

When: 1986

Explosions are bad, but why is this one such a big deal?

The nuclear radiation levels released from the explosion were enormous. Worked were exposed to lethal doses in just one minute.



#### Radioactive Decay: Chernobyl disaster

Why is it still uninhabited?

The model represents the amount of plutonium *P* that remains after *t* years after an initial amount *i*:

 $P = i \left(\frac{1}{2}\right)^{t/24,360}$ 

When will *P* be 0? Never...

Experts believe that, at the actual explosion site, it will be safe for humans in 20,000 years.

#### Rules of Exponents or Laws of Exponents

Multiplication Rule	$a^x \times a^y = a^{x+y}$	
Division Rule	$a^x \div a^y = a^{x-y}$	
Power of a Power Rule	$\left(a^{x}\right)^{y}=a^{xy}$	
Power of a Product Rule	$(ab)^x = a^x b^x$	
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	
Zero Exponent	$a^{\circ} = 1$	
Negative Exponent	$a^{-x} = \frac{1}{a^x}$	
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$	

## Examples: Simplify.

1)  $2m^{-3}n^{-1}p^3 \cdot 4nm^3p^2 = 8p^5$ 

2) 
$$2x^{-1}y^{-1} \cdot yx^4 z^{-4} = \frac{2x^3}{z^4}$$

Examples: Simplify.  
3) 
$$(2a^{-3}b^{-2}c^4)^4 = \frac{16c^{16}}{a^{12}b^8}$$

4) 
$$(2x^{-2}z^{3})^{-3} = \frac{x^{6}}{8z^{9}}$$

# Examples: Simplify.

5) 
$$\frac{2m^{-4}p^2}{mp^4q^2} = \frac{2}{m^5p^2q^2}$$

6) 
$$\frac{2r^{-3}}{4p^4q^{-4}} = \frac{q^4}{2r^3p^4}$$

Examples: Simplify.  
7) 
$$(2x^{-4}y^{-2}z^{3})^{-4} \cdot y^{2}z^{3} = \frac{x^{16}y^{10}}{16z^{9}}$$

8) 
$$\left(\frac{x^3y^{-1}z^4}{(2z^3)^2}\right)^4 = \frac{x^{12}}{256y^4z^8}$$





# Examples: Simplify.

9) 
$$\frac{x^5 + x^{-2}}{x^{-3}} = \frac{x^5}{x^{-3}} + \frac{x^{-2}}{x^{-3}} = x^8 + x$$

10) 
$$\frac{x^5 \cdot x^{-2}}{x^{-3}} = \frac{x^3}{x^{-3}} = x^6$$

## Practice Problems

5-1: Page 173 #5, 7, 11, 13, 17, 21-32

